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**ANALYTICAL AND EXPERIMENTAL INVESTIGATION OF THE
EFFECT OF TUNED VISCOELASTIC DAMPERS ON RESPONSE
OF SIMPLE BEAMS WITH VARIOUS BOUNDARY CONDITIONS**

D. I. G. JONES

TECHNICAL REPORT AFML-TR-67-214

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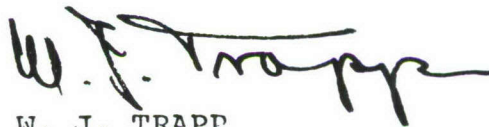
FOREWORD

This report was prepared by the Strength and Dynamics Branch, Metals and Ceramics Division, under Project No. 7351, "Metallic Materials," Task No. 735106, "Behavior of Metals". This research work was conducted in the Air Force Materials Laboratory, Research and Technology Division, Wright-Patterson Air Force Base, Ohio by Dr. D. I. C. Jones.

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This technical report has been reviewed and is approved.



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ABSTRACT

An approximate analysis of the response in the fundamental mode of any simple single span beam with tuned viscoelastic dampers attached at discrete locations to a harmonic loading with arbitrary spatial distribution is derived. It is shown that, to a good degree of approximation, a single expression can be made to represent the response in the fundamental mode of a beam with any boundary conditions, provided that certain effective mass and stiffness parameters are defined for the beam-damper configuration. Comparisons are made with experiments and with an exact theory, subject to the limitations of the Euler-Bernoulli beam equation, of the response and damping of a cantilever beam having an isolated harmonically varying load at the free end and a clamped-clamped beam, with a tuned damper at the center, under shaker excitation. Good agreement between the exact and approximate theories and the experiments is demonstrated. Conclusions are drawn concerning the equivalent damping introduced into the simple structure by the tuned dampers and the damper natural frequency needed for optimal damping.

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LIST OF SYMBOLS

A	Resonant amplification factor (shaker excitation)
exp	Exponential function
E	Young's Modulus for beam material
F_j	Force transmitted back to structure by j th damper unit
i	Square root of minus 1
I	Second moment of area of beam cross section
J	Total number of dampers on beam
k	Real part of stiffness of damper unit
L	Length of beam
m	Tuning mass of damper unit. Also dummy subscript
$P(x)$	Amplitude of transverse applied loading
P_n	n th term in expansion of $P(x)$ as series of normal modes
Q	Resonant amplification factor (force excitation)
t	Time
$W(x)$	Amplitude of transverse displacement of beam
W_n	n th term in expansion of $W(x)$ as series of normal modes
x	Station along beam
x_j	Station of j th damper
α	See equation (22)
β	See equation (23)
γ	See equation (24)
Γ	$kL^3/EI\xi_1^4$ - non-dimensional stiffness parameter
Γ_e	See equation (12)
δ	Dirac Delta function
Δ	x/L

Δ_j	x_j/L
η	Loss factor of viscoelastic spring of damper unit
η_s	Effective loss factor of beam-damper configuration [= $(Q^2-1)^{-1/2}$]
μ	Mass per unit length of beam
ξ	$(\mu\omega^2L^4/EI)^{1/4}$ - frequency parameter
ξ_n	$(\mu\omega_n^2L^4/EI)^{1/4}$ - n th eigenvalue of undamped beam
ϕ_n	n th normal mode of undamped beam
ψ	$m/\mu L$ - non-dimensional mass parameter
ψ_e	See equation (11)
ω	Circular frequency
ω_n	Natural circular frequency of n th normal mode of undamped beam
ω_D	$= (k/m)^{1/2}$ - natural frequency of damper unit

Suffixes:

D	subscript referring to damper
e	subscript referring to reduction of data for all boundary conditions
j	subscript denoting j th damper
n	subscript referring to number of normal mode
s	subscript referring to effective damping for beam-damper system

I. INTRODUCTION

The tuned damper, consisting of a spring-dashpot combination (or a viscoelastic spring) connecting a mass to a point on a vibrating structure, has recently been examined from the point of view of a possible application to the damping of complex structures exhibiting closely spaced resonant frequencies [1, 2]. Since analysis is usually difficult in such cases investigations of the effect of tuned dampers on the response of simpler structures have served as essential preliminaries. Such analyses have been carried out by Snowdon [3] and others [4-7].

However, although exact solutions within the framework of the Euler-Bernoulli equation have been obtained for several beam-damper configurations, under various harmonic loadings, no attempt has apparently been made to derive a general theory, applicable equally to all simple beam structures and harmonic loadings. Such a theory is developed in this paper. An approximate theory, applicable for the fundamental mode primarily, is obtained for a simple beam with tuned dampers at various points and subjected (i) to a harmonic loading of arbitrary spatial dependence and (ii) to displacement (shaker) excitation at the support(s). It is shown that a single expression can be made to represent the transmissibility for all boundary conditions. Comparisons are made with exact solutions obtained for a cantilever beam, harmonically loaded by a force at the free end, and with a tuned damper at the free end [4] and for a clamped-clamped beam with a tuned damper at the center, subjected to displacement (shaker) excitation at the supports [5]. The exact and approximate theories are shown to be in good agreement.

Experimental investigations of cantilever and clamped-clamped beams, with an isolated tuned damper at the free end and at the center respectively, are described. It is shown that the main conclusions of the various theories are borne out.

II. APPROXIMATE ANALYSIS OF TUNED DAMPERS ON SINGLE SPAN BEAM UNDER FORCE EXCITATION

Consider a single span beam of length L with tuned visco-elastic dampers of complex spring stiffness $k(1+i\eta)$ and mass m at a number of points $x = x_j$ ($j = 1$ to J) as in Figure 1. The amplitude of the harmonic j force transmitted back to the structure (F_j) by the damper at the point x_j is readily obtained [4, 5] by solving the equation of motion of the mass m for the damper subjected to a harmonic input displacement of amplitude $W(x_j)$ at the point of attachment to the beam. Then:

$$F_j = \frac{-m\omega^2 W(x_j) \delta(x - x_j)}{1 - m\omega^2 / k(1 + i\eta)} \quad (1)$$

The Euler-Bernoulli equation for the beam under the action of a harmonic loading of amplitude $P(x)$ is therefore written:

$$EI(d^4W/dx^4) - \mu\omega^2 W - \frac{m\omega^2}{1 - m\omega^2 / k(1 + i\eta)} \sum_{j=1}^J W(x_j) \delta(x - x_j) = P(x) \quad (2)$$

If $W(x)$ and $P(x)$ are now expanded as series of normal modes of the undamped beam, assumed to be known, then these modes must satisfy the homogeneous equation of motion;

$$d^4\phi_n(x)/dx^4 - (\mu\omega_n^2/EI) \phi_n(x) = 0 \quad (3)$$

If use is made of this fact, equation (2) may be written:

$$\begin{aligned} \sum_{n=1}^{\infty} (\mu\omega_n^2 - \mu\omega^2) W_n \phi_n(x/L) - \frac{m\omega^2}{1 - m\omega^2 / k(1 + i\eta)} \sum_{j=1}^J \delta(x - x_j) \sum_{m=1}^{\infty} W_m \phi_m(x_j/L) \\ = \sum_{n=1}^{\infty} P_n \phi_n(x/L) \end{aligned} \quad (4)$$

$$\text{where } W(x) = \sum_{n=1}^{\infty} W_n \phi_n(x/L) \quad (5)$$

$$\text{and } P(x) = \sum_{n=1}^{\infty} P_n \phi_n(x/L) \quad (6)$$

If we now factor all the terms of equation (4) by $\phi_n(x/L)$ and integrate with respect to x from 0 to L , then:

$$(\mu\omega_n^2 - \mu\omega^2)W_n \int_0^L \phi_n^2(x/L) dx - P_n \int_0^L \phi_n^2(x/L) dx - \frac{m\omega^2}{1-m\omega^2/k(1+i\eta)} \sum_{j=1}^J \phi_n(x_j/L) \sum_{m=1}^{\infty} W_m \phi_m(x_j/L) = 0 \quad (7)$$

use being made of the orthogonal property of the normal modes i.e.

$$\int_0^L \phi_m(x/L) \phi_n(x/L) dx = 0 \quad (m \neq n) \quad (8)$$

In this particular investigation, one is interested primarily in the response in the vicinity of the fundamental mode of the beam, for which $n = 1$. Therefore:

$$W_1 = \frac{P_1 L^4 / EI}{\xi_1^4 - \xi^4 - \frac{\psi \xi^4 \sum_{j=1}^J \phi_1(\Delta_j) \sum_{m=1}^{\infty} (W_m / W_1) \phi_m(\Delta_j)}{[1 - \psi \xi^4 / \Gamma(1+i\eta)] \int_0^1 \phi_1^2(\Delta) d\Delta}} \quad (9)$$

It is clear from equation (7) that, since for any simple beam $\omega_n^2 \gg \omega_1^2$ ($n > 1$), $W_n \ll W_1$ in the vicinity of the first resonant frequencies of the beam-damper system. Therefore only the first term in the series with respect to m in equation (9) need be retained so that, at the point of maximum amplitude where $\phi_1=1$,

$$\frac{EI \xi_1^4 W}{P_1 L^4} = \frac{1}{1 - (\xi/\xi_1)^4 - \frac{\psi_e (\xi/\xi_1)^4}{1 - \psi_e (\xi/\xi_1)^4 / \Gamma_e(1+i\eta)}} \quad (10)$$

where ψ_e is an effective mass parameter defined by

$$\psi_e = \psi \sum_{j=1}^J \phi_1^2(\Delta_j) / \int_0^1 \phi_1^2(\Delta) d\Delta \quad (11)$$

and
$$\Gamma_e = \Gamma \sum_{j=1}^J \phi_1^2(\Delta_j) / \int_0^1 \phi_1^2(\Delta) d\Delta \quad (12)$$

is an effective stiffness parameter. It has therefore been shown that the theory of the response of any simple beam, for which the resonant frequencies are sufficiently well separated for certain approximations to be made, can be reduced to a single expression if appropriate effective mass and stiffness parameters are defined for each particular set of boundary conditions. Certainly, the integrals and summations in equations (11) and (12) are readily evaluated for most cases using the tables of normal modes given by Bishop and Johnson [8]. Some of the integrals and summations are given in Table A for a number of boundary conditions.

It will be seen that equation (10) is the same as that obtained if one had assumed that the beam was uniformly covered by a distribution of tuned dampers with the effective mass parameter ψ_e now representing the true mass ratio for the distributed dampers i.e. the ratio of the total mass of the dampers to the total mass of the beam [9]. This is apparent from equation (11) when j approaches infinity. Similarly, Γ_e is seen to be equal to $L^3/EI\xi_1^4$ times the total stiffness of all the damper springs in parallel. The theory of the beam with distributed tuned dampers has already been developed [9, 10].

On the basis of equation (10), the amplitude $|W|$ of the response can readily be determined for various specific values of ψ_e , Γ_e and η as a function of ξ/ξ_1 or $(\xi/\xi_1)^2$. Typical graphs of $(EI\xi_1^4/P_1L^4) |W|$ are plotted against $(\xi/\xi_1)^2$, which is proportional to the frequency ω , in Figures 2 and 3. Further data and graphs are available [9] for other values of η .

From the response spectra which have essentially two resonant peaks, a measure of the performance of the dampers in damping the beam is given by an arbitrarily defined effective loss factor η_s , defined by $\eta_s = (Q^2 - 1)^{-1/2}$, where Q is the amplification factor of each resonant peak. Computed values of Q at resonance are given in Table II. Typical graphs of η_s against the effective stiffness parameter Γ_e are plotted in Figure 4 for the high and low frequency resonant peaks. Further data for other values of ψ_e are available [9].

At the point where the two resonance peaks are of equal amplitude, the dampers are said to be optimally tuned [3, 6]. This is the point at which the curves of η_s against Γ_e cross over in Figure 4. At all other values of Γ_e , one or other of the two resonance peaks will have a higher amplification factor Q than at the point of optimal tuning. Typical graphs of the value of η_s for optimally tuned dampers are plotted in Figure 5 against the parameter ψ_e . The data is taken from Table III, where values of η_s and Γ_e are given for various ψ_e and η for both the exact theory (discussed later) and the present approximate theory. These tabulated values are taken from graphs such as Figure 4. A cross plot of the data given in Figure 5 gives η_s as a function of η for various ψ_e , and this data is plotted in Figure 6.

The values of Γ_e at which the dampers are optimally tuned are also of great interest since, from the definition of Γ_e :

$$\begin{aligned}\Gamma_e &= (kL^3/EI\xi_1^4) \sum_{j=1}^J \phi_1^2(\Delta_j) / \int_0^1 \phi_1^2(\Delta) d\Delta \\ &= (\omega_D/\omega_1)^2 \psi_e \\ \omega_D/\omega_1 &= (\Gamma_e/\psi_e)^{1/2}\end{aligned}\tag{13}$$

It is, therefore, a simple matter to determine the ratio of the natural frequency ω_D of the damper to the natural frequency ω_1 of the undamped beam from the values of Γ_e at the point of optimal tuning. A typical graph of ω_D/ω_1 against ψ_e is shown in Figure 7. Of more interest, however, is the graph of $(\omega_D/\omega_1)(1+\psi_e)^{1/2}(1+\eta^2)^{1/4}$, for the exact and approximate theories, against ψ_e plotted for several values of the damper loss factor η in Figure 8. This empirically derived representation collapses all the data on to a single straight line so that the relationship between ω_D/ω_1 and ψ_e and η is:

$$\omega_D/\omega_1 = (1+\psi_e)^{-1/2} (1+\eta^2)^{-1/4}\tag{14}$$

Equation (14) implies that, if ψ_e and η are known, it is possible to determine the natural frequency ω_D of the damper such that the beam-damper system is optimally damped. This simple relationship should therefore be of value for simple systems exhibiting widely separated resonance frequencies and may serve as a guide for more complex structures to which it is desired to attach tuned viscoelastic dampers (See [2]).

III. APPROXIMATE ANALYSIS OF TUNED DAMPERS ON SINGLE SPAN BEAM UNDER SHAKER EXCITATION

If U is the amplitude of transverse displacement of any point x of the beam relative to the clamped end or ends (the analysis must clearly be limited to cases where at least one end of the beam is attached to the shaker), the equation of motion may be written:

$$EI(d^4U/dx^4) - \mu\omega^2[U+X] - \frac{m\omega^2}{1-m\omega^2 / k(1+i\eta)} \sum_{j=1}^J [U(x_j)+X] \delta(x-x_j) = 0 \quad (15)$$

which may also be written:

$$\begin{aligned} EI(d^4U/dx^4) - \mu\omega^2 U - \frac{m\omega^2}{1-m\omega^2 / k(1+i\eta)} \sum_{j=1}^J U(x_j) \delta(x-x_j) \\ = \mu\omega^2 X + \frac{m\omega^2 X}{1-m\omega^2 / k(1+i\eta)} \sum_{j=1}^J \delta(x-x_j) \end{aligned} \quad (16)$$

This equation is clearly different from equation (2) but may be solved in much the same way. Again, we replace the response $U(x)$ by the appropriate expansion in normal modes. Then:

$$U(x) = \sum_{n=1}^{\infty} U_n \phi_n(x/L) \quad (17)$$

and, using equation (3) which applies equally to this case, equation (16) becomes:

$$\begin{aligned}
(\xi_n^4 - \xi^4) \sum_{n=1}^{\infty} U_n \phi_n(x/L) &= \frac{m\omega^2 L^4 / EI}{1 - m\omega^2 / k(1+i\eta)} \sum_{m=1}^{\infty} U_m \phi_m(x_j/L) \sum_{j=1}^J \delta(x - x_j) \\
&= \xi^4 X + \frac{m\omega^2 L^4 / EI}{1 - m\omega^2 / k(1+i\eta)} \sum_{j=1}^J \delta(x - x_j) \quad (18)
\end{aligned}$$

If we factor both sides of equation (18) by $\phi_n(x/L)$, integrate from 0 to L with respect to x and make use of orthogonal property of the normal modes:

$$\begin{aligned}
(\xi_n^4 - \xi^4) U_n \int_0^1 \phi_n(\Delta) d\Delta &= \frac{\psi \xi^4}{1 - \psi \xi^4 / \Gamma(1+i\eta)} \sum_{m=1}^{\infty} U_m \phi_m(\Delta_j) \sum_{j=1}^J \phi_n(\Delta_j) \\
&= \xi^4 X \int_0^1 \phi_n(\Delta) d\Delta + \frac{\psi \xi^4}{1 - \psi \xi^4 / \Gamma(1+i\eta)} \sum_{j=1}^J \phi_n(\Delta_j) \quad (19)
\end{aligned}$$

Considering the first mode only, therefore:

$$\begin{aligned}
U_1 \left[\xi_1^4 - \xi^4 - \frac{\psi \xi^4 \sum_{j=1}^J \phi_1(\Delta_j) \sum_{m=1}^{\infty} (U_m/U_1) \phi_m(\Delta_j)}{[1 - \psi \xi^4 / \Gamma(1+i\eta)] \int_0^1 \phi_1^2(\Delta) d\Delta} \right] \\
= X \frac{\xi^4 \int_0^1 \phi_1(\Delta) d\Delta}{\int_0^1 \phi_1^2(\Delta) d\Delta} + \frac{\psi \xi^4 \sum_{j=1}^J \phi_1(\Delta_j)}{1 - \frac{\psi \xi^4}{\Gamma(1+i\eta)} \int_0^1 \phi_1^2(\Delta) d\Delta} \quad (20)
\end{aligned}$$

and, since $U_n \ll U_1$ in the neighborhood of the fundamental frequency, we may write as an approximation:

$$\frac{U_1}{X} = \frac{(\xi/\xi_1)^{4\alpha} + \frac{\psi(\xi/\xi_1)^{4\beta}}{1 - \psi(\xi/\xi_1)^4/\Gamma(1+i\eta)}}{1 - (\xi/\xi_1)^4 - \frac{\psi(\xi/\xi_1)^{4\gamma}}{1 - \psi_e(\xi/\xi_1)^4/\Gamma_e(1+i\eta)}}$$

$$\frac{W}{X} = \frac{U_1 + X}{X} \left[1 + (\xi/\xi_1)^4 \frac{\left[\frac{\alpha + \beta \psi(\xi/\xi_1)^4 / \{1 - \psi(\xi/\xi_1)^4/\Gamma(1+i\eta)\}}{1 - (\xi/\xi_1)^4 - \gamma \psi(\xi/\xi_1)^4 / [1 - \psi(\xi/\xi_1)^4/\Gamma(1+i\eta)]} \right]}{1} \right] \quad (21)$$

$$\text{where } \alpha = \int_0^1 \phi_1(\Delta) \phi_\Delta / \int_0^1 \phi_1^2(\Delta) d\Delta \quad (22)$$

$$\text{and } \beta = \sum_{j=1}^J \phi_1(\Delta_j) / \int_0^1 \phi_1^2(\Delta_j) d\Delta \quad (23)$$

$$\text{and } \gamma = \sum_{j=1}^J \phi_1^2(\Delta_j) / \int_0^1 \phi_1^2(\Delta) d\Delta \quad (24)$$

It is seen that the response is now governed by two additional parameters, namely α and β . In the special case where $J \rightarrow \infty$, i.e. the dampers are uniformly distributed, $\alpha \rightarrow \beta$. Also, for $J=1$ and $\phi_1(\Delta_j)=1$, i.e. the case where the single damper location and the point at which the mode shape is normalized are identical, then $\beta=\gamma$ also. This particular case is of some importance and analysis will be limited to this case. If $\beta=\gamma$, therefore:

$$\frac{W}{X} = 1 + \left(\frac{\xi}{\xi_1} \right)^4 \left[\frac{\alpha + \psi_e (\xi/\xi_1)^4 / [1 - \psi_e (\xi/\xi_1)^4 / r_e (1 + i\eta)]}{1 - (\xi/\xi_1)^4 - \psi_e (\xi/\xi_1)^4 / [1 - \psi_e (\xi/\xi_1)^4 / r_e (1 + i\eta)]} \right]$$

$$= \frac{1 + (\xi/\xi_1)^4 (\alpha - 1)}{1 - (\xi/\xi_1)^4 - \frac{\psi_e (\xi/\xi_1)^4}{1 - \psi_e (\xi/\xi_1)^4 / r_e (1 + i\eta)}} \quad (25)$$

In this particularly simple case, therefore, the problem of determining $|W/X|$ under shaker excitation reduces to that of factoring the response under force excitation, given in equation (10) by $1 + (\xi/\xi_1)^4 (\alpha - 1)$. For the response determined in this way, two peaks are again observed and it has been shown [4, 5] that, for shaker excitation, the effective loss factor η_s is defined by the relationship:

$$\eta_s = \frac{\alpha \phi_1(\Delta)}{\sqrt{A^2 - 1}} \quad (26)$$

where A is the amplification factor i.e. the value of $|W/X|$ at each resonance in the fundamental modes. Values of the amplification factor A under shaker excitation are given in Table II for a clamped-clamped beam along with values of Q for Force excitation. Typical graphs of η_s defined as in equation 25, against r_e are shown in Figure 9. From these graphs, the optimum loss factor corresponding to the point of cross-over, can be read off and plotted against ψ_e for values of η . The points are plotted in Figure 5 and show that the variation of η_s with ψ_e is practically independent of whether the beam is force or shaker excited.

On the other hand, graphs of $(r_e/\psi_e)^{1/2} (1 + \psi_e)^{+1/2} (1 + \eta^2)^{+1/4}$ and $\omega_D/\omega_1 = (r_e/\psi_e)^{1/2}$ against ψ_e do show some differences, as figures 10 and 11 show.

IV. COMPARISON OF EXACT AND APPROXIMATE ANALYSES

(i) Cantilever beam under force excitation

Previous investigations of tuned dampers on simple beams have led to exact solutions of the Euler-Bernoulli Equation for a beam with a tuned damper at an antinodal point. For example, the response of a cantilever beam with a tuned damper at the free end is described by Young [11] and Nashif [4]. Graphs of $(EI/PL^4) |W|$ against ξ for a load of amplitude P at the free end were obtained from the exact theory [4] and were shown to consist of two resonance peaks in the vicinity of the fundamental mode, as in the approximate theory. Graphs of η_s against Γ were drawn as for the approximate theory and some of the results are tabulated in Table III for the optimally tuned case where the two resonance peaks are of equal amplitude. Since only one damper was considered for the exact theory [5]

of the cantilever beam, $\sum_{j=1}^J \phi_1^2(\Delta_j) = 1$ and, as in Table I,

$\int_0^1 \phi_1^2(\Delta) d\Delta = 0.25$. Therefore, for this case, $\psi_e = 4\psi$ and

$\Gamma_e = 4\Gamma$. From the values of Γ and ψ [4] therefore, Γ_e and ψ_e were derived and the values of η_s plotted against ψ_e in Figure 5. It is seen that the computed points lie essentially along the same line as given by the approximate theory.

Furthermore, the values of $(\omega_D/\omega_1)(1+\psi_e)^{1/2}(1+\eta^2)^{1/4}$, when plotted against ψ_e , lie on the same straight line as given by the approximate theory, as in Figure 8.

(ii) Clamped-clamped beam under shaker excitation

A previous investigation [5] has given the exact theory of a tuned damper at the center of a clamped-clamped beam on the response under shaker excitation. Some of the results

are tabulated in Tables II and IV. Again $\sum_{j=1}^J \phi_1^2(\Delta_j) = 1$ and, as in Table I, $\int_0^1 \phi_1^2(\Delta) d\Delta = 0.439$. Thus, for this case,

$\psi_e = 2.086\psi$ and $\Gamma_e = 2.086\Gamma$. From the values of Γ and ψ [5], Γ_e and ψ_e were deduced and entered into Table V and graphs of η_s plotted against ψ_e , as in Figure 5. It is seen that the computed points lie along the same curve as all the others.

Values of ω_D/ω_1 and $(\Gamma/\psi\xi_1^4)^{1/2}(1+\psi_e)^{1/2}(1+\eta^2)^{1/4}$ when plotted against ψ_e , lie along the same line as given by the approximate theory in Figures 10 and 11 respectively.

IV. EXPERIMENTAL VERIFICATION

(i) Cantilever beam with distributed tuned dampers under shaker excitation

This investigation has previously been reported in reference [9]. In brief, a cantilever beam with eleven tuned dampers of the geometry shown in Figure 12 was vibrated by an electrodynamic shaker. An accelerometer at the tip was used to measure the response and the effective damping deduced from the appropriate relationship, namely $\eta_s = 1/\sqrt{A^2-1}$.

The length of the beam was varied so as to obtain proper tuning, i.e. to make the two response peaks, corresponding to the fundamental mode of equal amplitude. The loss factor of each damper was determined as in [9] and plotted in Figure 13. Comparison of the measured values of η_s for $\eta = 0.175$ and 0.09 and various values of ψ_e are shown^s in Figure 14.

It is seen that the agreement between theory and experiment is good.

Another part of the investigation, not previously reported, involved the verification of the relationship given in equation (14) for the point of optimum tuning. The geometry of the dampers used in this investigation is shown in Figure 15 A. The density of the aluminum was 0.101 Lb/in³ and that of the viscoelastic material (LD-400) was 0.0522 Lb/in³. The total weight of the resilient part of the damper up to the last half inch in which the tip mass is situated is:

$$\begin{aligned} m_D &= 0.5 \times 1 \times 0.02 \times 0.101 \times 454 \\ &+ 0.0522 \times 0.5 \times 0.75 \times \tau \times 454 \\ &= 0.459 + 8.83\tau \text{ gms} \end{aligned}$$

where τ is the thickness of the viscoelastic material in inches and the damper breadth is 0.5 inches. Now an additional mass equal to the weight of the outermost half inch of the damper beam must be included. This amounts to $0.5 \times 0.5 \times 0.02 \times 0.101 \times 454 \times 0.23$ gms. The total effective mass to be added to the nominal mass m_t at the free end in order to give the true mass can be shown from [4] to be

$$\begin{aligned} m &= m_t + 0.23 + 0.236 m_D \text{ gms} \\ &= m_t + 0.338 + 2.08\tau \text{ gms} \end{aligned}$$

$\omega_D m^{1/2}$ should depend only on τ/h_D for this particular geometry of damper. Tests were carried out for these dampers, with

various masses m_t at the free end under shaker excitation. An accelerometer was placed at the free end and formed part of the mass m_t . Response spectra showing the variation of the acceleration at the free end with frequency for several input accelerations at the clamped end were measured and the frequency at which the output acceleration was greatest noted. Some typical results are shown in Table V. Further tests, in which the response at the tip was measured optically were also carried out, and the results are given in Table VI. From these results, a graph of $\omega_D m^{1/2}$ against τ/h_D was plotted as in Figure 16. It is seen that the data do indeed collapse on to a single curve.

Since optimal tuning was obtained by varying the length of the beam until the two response peaks corresponding to the fundamental mode were of equal amplitude, the fundamental natural frequency of the cantilever beam with no dampers (but with the bolts used to hold the clamps in place) was measured on the shaker and plotted against the beam length L , as in Figure 17.

Table 1 of reference [9] gives the experimental data obtained for the clamped-free beam with eleven distributed dampers under shaker excitation. From the values of L for optimal tuning, ω_1 can be read off Figure 16 and from the values of m_t (referred to as "m" in Table 1 of reference [9]) we can deduce ω_D . Hence $(\omega_D/\omega_1) \sqrt{1+\psi_e} (1+\eta^2)^{1/4}$ can be calculated for the point of optimum tuning. This calculation is carried out in Table VII. The values of $(\omega_D/\omega_1) \sqrt{1+\psi_e} (1+\eta^2)^{1/4}$ are plotted against ψ_e in Figure 18.

(ii) Cantilever beam with tuned damper at free end under shaker excitation

This investigation has previously been reported in reference [4]. In this experimental investigation, the effective damping of the setup shown in Figure 19 was determined from the experimental amplification factor A under shaker excitation by means of the relationship given in Equation 26. The damper configuration used is shown in Figure 15B. The length of the beam was varied so as to obtain proper tuning in the fundamental mode. For the point of optimum damping, graphs of the optimum η_s against ψ were plotted, as in reference [4]. These graphs are re-plotted as graphs of η_s against ψ_e for $\eta = 0.22$ and $\eta = 0.8$ in Figures 20 and 21 respectively. It is seen that the agreement between theory and experiment is satisfactory. The loss factor η of each damper was determined as in [4] and is plotted against τ/h_D in Figure 22.

Measurements of the natural frequencies of the dampers for various tip masses m_t were again made. The value of m_D is, now

$$\begin{aligned} m_D &= 0.5 \times 1 \times 0.02 \times 0.101 \times 454 \\ &+ 0.0522 \times 0.5 \times 1 \times \tau \times 454 \\ &+ 0.459 + 11.8\tau \text{ gms} \end{aligned}$$

where τ is again the viscoelastic material thickness in inches.

$$\begin{aligned} m &= m_t + 0.23 + 0.236 m_D \\ &= m_t + 0.338 + 2.78\tau \text{ gms} \end{aligned}$$

as for the dampers used for the beam with distributed tuned dampers (case A). Again, graphs of $\omega_D m^{1/2}$ against τ/h_D were plotted on the basis of measured values of ω_D for various m_t . This data is given in Table VIII. It is seen that the points on the graph of $\omega_D m^{1/2}$ against τ/h_D in Figure 16 lie along the same line as for the dampers in case A, as would be expected. The small additional amount of viscoelastic material near the root of the cantilever damper contributes greatly to the damping but not to the damped natural frequency.

From Table 4 of Reference [4], the values of the test beam length L are obtained for the point of optimal tuning. A graph of ω_1 against L was obtained experimentally for the beam with a mass of 22 gms at the free end, and plotted in Figure 23. This mass represented the metal stamp used to ensure proper attachment of the cantilever damper at the free end of the test beam. Values of $(\omega_D/\omega_1) \sqrt{1+\psi} (1+\eta^2)^{1/4}$ were then obtained from the test data, as in Table IX, and plotted against ψ_e in Figure 18.

(iii) Clamped-clamped beam with tuned damper at center under shaker excitation

This investigation has previously been reported in reference [5]. In this experimental investigation the effective damping was determined from the observed resonance amplification factor by the relationship $\eta_s = 1.32(A^2 - 1)^{-1/2}$. This setup is illustrated in Figure 24^s, 25. The beam length was now fixed at 19.9 inches, with a fundamental frequency of 90 cps. Graphs of η_s against the mass ratio ψ for several

values of η are given in reference [5] and are re-drawn as graphs of η_s against ψ_e ($= 2.5\psi$) in Figures 20 and 21, for $\eta = 0.22$ and 0.8 respectively. It is seen that the collapsed data is in good agreement with the theory and with the data for the cantilever beam.

The damper used in this investigation is illustrated in Figure 15C. Measurements of the loss factors and natural frequencies of the dampers were again made. The loss factor measurements are plotted in Figure 22. The value of m_D is now:

$$\begin{aligned} m_D &= 0.101 \times 1 \times 0.063 \times 454 \ell_D \\ &+ 0.0522 \times 1 \times \ell_D \times \tau \times 454 \\ &= (2.83 + 23.2\tau) \ell_D \quad \text{gms} \end{aligned}$$

where τ is the thickness of the viscoelastic material in inches, ℓ_D is the damper length in inches, the damper breadth is 1 inch. Graphs of $\omega_D m^{1/2} \ell_D^{3/2}$ were again plotted against τ/h_D , on the basis of measured values of ω_D , in Figure 26. This data is given in Table X. The data for $\ell_D = 2.2$ inches, 2.7 inches and 3.7 inches all fall on the same curve.

From Table 2 of Reference [5], the values of the damper mass m_t needed for optimal tuning are taken and values of $(\omega_D/\omega_1) \sqrt{1+\psi_e} (1+\eta^2)^{1/4}$ calculated as in Table XI. The data is plotted in Figure 18.

VI. CONCLUSIONS

A close approximation to the response of a single span beam with any boundary conditions, having isolated tuned viscoelastic dampers at arbitrary locations, under the action of any harmonically varying loading has been derived. Effective mass and stiffness parameters, and a system loss factor are defined. Comparisons are made with an exact theory of the response and damping of a clamped cantilever beam with a single tuned damper and an isolated harmonic force at the free end and a clamped-clamped, beam under shaker excitation, with a tuned damper at the center.

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TABLE 1

STANDARD INTEGRALS FOR VARIOUS BEAM CONFIGURATIONS

Boundary Conditions	Clamped - Free	Pinned- Pinned	Clamped- Pinned	Clamped- Clamped	Free- Free
ξ_1	1.875	3.142	3.927	4.730	4.730
ξ_1^4	12.36	97.4	237.7	500.6	500.6
Δ_j	1.00	0.50	0.50	0.50	0.50
$\phi_1(\Delta_j)$	1.000	1.000	0.957	1.000	1.000
$\int_0^1 \phi_1(\Delta) d\Delta$	0.392	0.637	0.570	0.523	0.000
$\int_0^1 \phi_1^2(\Delta) d\Delta$	0.250	0.500	0.439	0.397	0.250
$\phi_1^2(\Delta_j) / \int_0^1 \phi_1^2(\Delta) d\Delta$	4.000	2.000	2.086	2.519	1.479
$\int_0^1 \phi_1(\Delta) d\Delta / \int_0^1 \phi_1^2(\Delta) d\Delta$	1.568	1.274	1.298	1.317	0.000

TABLE II

THEORETICAL RESONANT AMPLIFICATION FACTORS AND RESONANT
 FREQUENCIES FOR CLAMPED-CLAMPED BEAM ($\alpha-1 = 0.317$)

ψ_e	η	$r_e = \frac{\lambda}{\pi^4}$	Force Excitation				Shaker Excitation	
			Peak 1		Peak 2		Amplification A	
			Q	$(\xi/\xi_1)^4$	Q	$(\xi/\xi_1)^4$	Peak 1	Peak 2
.1	.2	.031	-	-	76.0	1.05	-	101
		.052	-	-	27.9	1.09	-	37.6
		.083	5.1	.66	7.5	1.24	6.15	10.5
		.124	14.9	.80	2.35	1.61	18.7	3.56
		.156	26.1	.83	-	-	33.0	-
.1	.5	.031	-	-	32.0	1.05	-	42.4
		.052	-	-	12.7	1.07	-	16.8
		.083	5.2	.825	-	-	6.5	-
		.124	11.7	.85	-	-	14.6	-
		.154	18.2	.87	-	-	23.2	-
.1	1	.031	-	-	18.6	1.03	-	24.6
		.052	-	-	9.7	1.01	-	12.8
		.083	10.0	.91	-	-	12.7	-
		.124	16.6	.89	-	-	21.0	-
		.154	22.6	.89	-	-	29.1	-
.1	1.5	.031	14.8	1.01	-	-	19.5	-
		.0103	53.2	1.01	-	-	70.2	-
		.0206	23.5	1.01	-	-	31.0	-
		.0412	11.8	.99	-	-	15.6	-
		.062	11.9	.93	-	-	15.4	-
.1	2	.0102	41.0	1.01	-	-	54.2	-
		.031	13.8	.99	-	-	18.1	-
		.072	17.3	.91	-	-	22.2	-
		.154	39.2	.91	-	-	51.0	-
.2	.2	.072	-	-	31.5	1.11	-	42.5
		.103	-	-	15.6	1.18	-	21.4
		.134	3.9	.52	8.8	1.28	4.5	12.4
		.165	5.9	.60	5.4	1.40	7.0	7.82
		.206	9.5	.66	3.2	1.61	11.5	4.85
		.268	16.2	.72	1.8	1.94	19.9	2.91

TABLE II (CONT'D)

ψ_e	η	$\Gamma_e = \frac{\lambda}{\pi^4}$	Force Excitation				Shaker Excitation	
			Peak 1		Peak 2		Amplification A	
			Q	$(\xi/\xi_1)^4$	Q	$(\xi/\xi_1)^4$	Peak 1	Peak 2
.2	.5	.072	-	-	13.2	1.09	-	17.80
		.103	-	-	6.9	1.16	-	9.50
		.134	3.2	.60	4.00	1.22	3.82	5.60
		.165	4.35	.66	2.61	1.30	5.25	3.70
		.268	10.3	.74	-	-	12.7	-
		.206	6.4	.70	-	-	7.82	-
		.310	13.2	.76	-	-	16.40	-
.2	1	.072	-	-	7.60	1.05	-	10.1
		.103	4.86	.99	-	-	6.40	-
		.134	4.99	.80	-	-	6.25	-
		.165	6.30	.78	-	-	7.85	-
		.310	14.7	.81	-	-	18.40	-
.2	1.5	.0414	-	-	11.8	1.03	-	15.7
		.072	6.50	.97	-	-	8.52	-
		.103	6.05	.87	-	-	7.72	-
		.136	7.30	.83	-	-	9.25	-
.2	2	.0414	9.70	1.01	-	-	12.8	-
		.072	6.80	.91	-	-	8.75	-
		.103	7.72	.85	-	-	9.82	-
		.134	9.70	.83	-	-	12.20	-
		.310	23.9	.85	-	-	30.40	-
.4	.2	.103	1.75	.21	29.0	1.13	1.86	39.5
		.155	2.46	.31	15.0	1.22	2.70	20.8
		.206	3.41	.39	9.25	1.33	3.80	13.2
		.258	4.76	.43	6.19	1.46	5.40	9.00
		.310	6.45	.49	4.40	1.60	7.50	6.70
		.392	9.70	.54	2.87	1.86	11.2	4.60
.4	.5	.103	1.50	.23	11.8	1.13	1.60	16.0
		.155	1.90	.33	6.30	1.22	2.07	8.75
		.206	2.47	.41	3.87	1.31	2.80	5.48
		.258	3.20	.48	2.62	1.42	3.70	3.81
		.310	4.06	.52	1.91	1.56	4.75	2.85
.4	1.0	.103	-	-	6.20	1.09	-	8.40
		.155	-	-	3.52	1.12	-	4.76
		.206	2.92	.58	-	-	3.47	-
		.258	3.74	.60	-	-	4.47	-
		.310	4.65	.62	-	-	5.55	-

TABLE II (CONT'D)

ψ_e	η	$\frac{\Gamma_e}{\lambda}$ π^4	Force Excitation				Shaker Excitation	
			Peak 1		Peak 2		Amplification A	
			Q	$(\xi/\xi_1)^4$	Q	$(\xi/\xi_1)^4$	Peak 1	Peak 2
.4	1.5	.072	-	-	6.95	1.05	-	9.30
		.103	4.55	1.03	-	-	6.05	-
		.155	3.26	.79	-	-	4.10	-
		.206	4.02	.66	-	-	4.88	-
.4	2	.072	5.56	1.01	-	-	7.20	-
		.103	4.00	.93	-	-	5.10	-
		.155	4.08	.72	-	-	4.95	-
		.206	5.20	.68	-	-	6.30	-
		.310	7.85	.68	-	-	9.55	-
.8	.2	.206	2.24	.206	16.4	1.26	2.38	23.0
		.290	3.14	.247	10.1	1.38	3.37	14.5
		.310	3.59	.268	8.60	1.42	3.90	12.4
		.352	4.03	.289	7.20	1.51	4.33	10.6
		.413	4.85	.330	5.65	1.61	5.36	8.60
		.454	5.50	.330	4.90	1.68	6.10	7.52
		.536	7.10	.370	3.81	1.85	7.90	6.07
.8	.5	.206	1.70	.186	6.20	1.26	1.80	8.70
		.290	2.14	.269	3.89	1.38	2.30	5.60
		.310	2.27	.288	3.50	1.42	2.47	5.10
		.352	2.54	.309	2.92	1.49	2.78	4.30
		.413	3.00	.330	2.31	1.61	3.32	3.50
		.454	3.33	.351	2.01	1.67	3.70	3.07
		.536	4.05	.392	1.58	1.84	4.55	2.52
.8	1	.206	1.74	.289	3.21	1.22	1.90	4.47
		.290	2.20	.350	2.05	1.30	2.45	2.90
		.310	2.32	.350	1.86	1.33	2.59	2.65
		.352	2.58	.391	1.57	1.38	2.90	2.25
		.413	3.03	.412	1.26	1.44	3.45	1.81
.8	1.5	.103	-	-	5.21	1.09	-	7.02
		.165	2.44	.99	-	-	3.20	-
		.206	2.05	.392	2.29	1.11	2.32	3.10
		.290	2.65	.435	-	-	3.03	-
		.352	3.16	.455	-	-	3.62	-
		.413	3.72	.475	-	-	4.30	-

TABLE II (CONT'D)

ψ_e	η	Γ_e $= \frac{\lambda}{\pi^4}$	Force Excitation				Shaker Excitation	
			Peak 1		Peak 2		Amplification A	
			Q	$(\xi/\xi_1)^4$	Q	$(\xi/\xi_1)^4$	Peak 1	Peak 2
.8	2	.103	-	-	4.02	1.05	-	5.35
		.165	2.44	.99	-	-	3.20	-
		.206	2.47	.475	-	-	2.84	-
		.290	3.25	.475	-	-	3.75	-
		.310	3.46	.495	-	-	4.02	-
		.352	3.90	.497	-	-	4.52	-
		.413	4.55	.497	-	-	5.28	-

TABLE III
THEORETICAL VALUES OF PARAMETERS FOR OPTIMAL TUNING
OF DAMPERS ON CANTILEVER BEAM UNDER FORCE EXCITATION

Loss Factor η	Approximate Theory					Exact Theory			
	ψ_e	η_s	Γ_e	$\sqrt{\frac{\Gamma_e}{\psi_e}}$	ψ_e	η_s	Γ_e	$\sqrt{\frac{\Gamma_e}{\psi_e}}$	
0.2	0.10	0.170	0.088	.94	0.08	0.165	0.073	.96	
0.5		0.204	0.083	.91		0.150	0.065	.90	
1.0		0.135	0.062	.79		0.110	0.055	.83	
2.0		0.076	0.040	.63		0.070	0.032	.63	
0.2	0.20	0.180	0.161	.90	0.20	0.185	0.161	.90	
0.5		0.270	0.150	.87		-	-	-	
1.0		0.250	0.112	.75		-	-	-	
2.0		0.150	0.068	.58		-	-	-	
0.2	0.40	0.190	0.290	.85	0.40	0.190	0.272	.83	
0.5		0.380	0.252	.80		0.375	0.240	.78	
1.0		0.420	0.200	.71		0.410	0.191	.69	
2.0		0.285	0.124	.56		0.290	0.136	.58	
0.2	0.60	0.190	0.372	.79	1.60	0.190	0.580	.60	
0.5		0.390	0.315	.73		0.435	0.520	.57	
1.0		0.480	0.237	.63		0.610	0.365	.68	
2.0		0.410	0.150	.50		0.680	0.227	.38	
0.2	0.80	0.210	0.445	.75	0.80	0.190	0.425	.73	
0.5		0.400	0.382	.69		0.405	0.366	.68	
1.0		0.510	0.282	.60		-	-	-	
2.0		0.505	0.176	.47		0.470	0.178	.47	

TABLE IV

THEORETICAL VALUES OF PARAMETERS FOR OPTIMAL TUNING OF
DAMPERS ON CLAMPED-CLAMPED BEAM UNDER SHAKER EXCITATION

Loss Factor η	Approximate Theory					Exact Theory		
	ψ_e	η_s	Γ_e	$\sqrt{\frac{\Gamma_e}{\psi_e}}$	ψ_e	η_s	Γ_e	$\sqrt{\frac{\Gamma_e}{\psi_e}}$
0.2	0.10	0.150	0.090	.950	0.10	0.165	0.091	.955
0.5		0.210	0.080	.895		0.202	0.080	.895
1.0		0.135	--	--		0.125	0.065	.868
1.5		0.095	--	--		0.095	0.050	.707
2.0		0.080	--	--		0.083	0.038	.616
0.2	0.20	0.185	0.167	.917	0.25	0.180	0.205	.910
0.5		0.310	0.148	.862		0.321	0.170	.830
1.0		0.280	0.116	.760		0.293	0.140	.750
1.5		0.200	0.088	.663		0.230	0.115	.678
2.0		0.160	0.070	.590		0.195	0.085	.582
0.2	0.40	0.190	0.290	.856	0.50	0.187	0.360	.850
0.5		0.380	0.250	.790		0.374	0.300	.775
1.0		0.430	0.190	.670		0.460	0.240	.692
1.5		0.400	0.160	.632		0.416	0.190	.616
2.0		0.350	0.111	.528		0.378	0.150	.550
0.2	0.80	0.200	0.490	.781	1.00	0.188	0.610	.782
0.5		0.420	0.410	.717		0.403	0.480	.692
1.0		--	--	--		0.580	0.350	.590
1.5		--	--	--		0.600	0.270	.520
2.0		0.550	0.200	.500		0.600	0.220	.469

TABLE V

LOSS FACTOR AND NATURAL FREQUENCY MEASUREMENTS FOR

TYPE A CANTILEVER DAMPERS

 $\ell_D = 1.125$ ins., $b = 0.50$ in., $h_D = 0.020$ ins.

τ ins	input g's	m_t gm	$\Delta m =$.338 + 2.08 τ (gm)	$m =$ $m_t + \Delta m$ (gm)	ω_D cps	$\omega_D \sqrt{m}$ cps (gm) ^{1/2}	output g's	A	$\eta =$ $1/\sqrt{A^2 - 1}$
0.02	0.5	9.5		9.9	57	180	4.9	9.8	.102
	0.5	11.0		11.4	45	152	5.6	11.2	.090
	0.5	11.0		11.4	46	155	8.1	16.2	.062
	0.5	11.0		11.4	46	156	8.9	17.3	.056
	0.5	14.0		14.4	45	171	8.5	17.0	.059
	0.5	18.5		18.9	39	169	8.4	16.8	.060
	0.5	27.5		27.9	29	153	6.1	12.2	.082
	2.0	18.5	0.4	18.9	38	165	23.0	11.5	.088
	2.0	23.0		23.4	34	164	21.2	10.6	.094
	2.0	27.5		27.9	30	158	19.0	9.5	.106
	5.0	5.0		5.4	78	181	36.0	7.2	.140
	5.0	9.5		9.9	57	180	38.0	7.6	.133
	5.0	14.0		14.4	43	163	37.0	7.4	.137
0.035	0.5	9.5		9.9	54	171	3.2	6.4	.158
	0.5	11.0		11.4	37	125	5.5	11.0	.092
	0.5	14.0		14.4	43	163	3.9	7.8	.129
	0.5	18.5		18.9	39	169	3.2	6.4	.158
	0.5	23.0	0.4	23.4	31	150	4.0	8.0	.126
	2.0	9.5		9.9	53	167	12.5	6.3	.160
	2.0	14.0		14.4	44	167	17.0	8.5	.119
	2.0	18.5		18.9	34	147	19.0	9.5	.106
	2.0	23.0		23.4	30	145	16.0	8.0	.126
	2.0	27.5		27.9	25	132	12.5	6.3	.160
	0.5	9.5		9.9	54	171	3.2	6.4	.158
	0.5	11.0		11.4	37	125	5.5	11.0	.092
	0.5	14.0		14.4	43	163	3.9	7.8	.129
	0.5	18.5		18.9	39	169	3.2	6.4	.158
	0.5	23.0	0.4	23.4	31	150	4.0	8.0	.126
	2.0	9.5		9.9	53	167	12.5	6.3	.160
	2.0	14.0		14.4	44	167	17.0	8.5	.119
	2.0	18.5		18.9	34	147	19.0	9.5	.106
	2.0	23.0		23.4	30	145	16.0	8.0	.126
	2.0	27.5		27.9	25	132	12.5	6.3	.160

TABLE V (CONT'D)

rins	input g's	m_t gm	$\Delta m =$.338 + 2.08 τ (gm)	$m =$ $m_t + \Delta m$ (gm)	ω_D cps	$\omega \sqrt{m}$ $\frac{D_{cps}}{(gm)^{1/2}}$	output g's	A	$\eta =$ $\frac{1}{1/\lambda^2 - 1}$
.062	0.5	9.5		10.0	56	177	-	-	-
	0.5	11.0		11.5	46	152	3.8	7.6	.132
	0.5	11.0		11.5	47	159	2.9	5.8	.175
	0.5	11.0		11.5	51	173	3.0	6.0	.169
	0.5	14.0		14.5	46	168	-	-	-
	0.5	18.5		19.0	38	165	-	-	-
	0.5	23.0		23.5	35	170	-	-	-
	0.5	27.5		28.0	31	164	-	-	-
	2.0	5.0	0.5	5.5	81	188	-	-	-
	2.0	9.5		10.0	54	170	-	-	-
	2.0	14.0		14.5	45	168	-	-	-
	2.0	18.5		19.0	39	170	-	-	-
	2.0	23.0		23.5	35	170	-	-	-
	2.0	27.5		28.0	31	164	-	-	-
0.125	0.5	9.5		10.1	89	283	2.15	4.3	.238
	0.5	11.0		11.6	46	157	3.2	6.4	.158
	0.5	11.0		11.6	50	171	3.2	6.4	.158
	0.5	18.5		19.1	56	245	2.0	4.0	.258
	0.5	23.0		23.6	49	238	2.05	4.1	.252
	0.5	27.5		28.1	44	233	2.0	4.0	.258
	2.0	9.5	0.6	10.1	84	266	10.5	5.3	.193
	2.0	14.0		14.6	68	260	9.2	4.6	.223
	2.0	18.5		19.1	51	223	8.8	4.4	.233
	2.0	23.0		23.6	46	224	9.1	4.6	.223
	2.0	27.5		28.1	41	217	8.85	4.42	.232
	0.5	9.5		10.1	89	283	2.15	4.3	.238
	0.5	11.0		11.6	46	157	3.2	6.4	.158
	0.5	11.0		11.6	50	171	3.2	6.4	.158
	0.5	18.5		19.1	56	245	2.0	4.0	.258

TABLE VI

OPTICAL DAMPING MEASUREMENTS FOR TYPE A TUNED DAMPERS

τ ins	input g's	m_t gm	$\Delta m =$.338 + 2.08 τ (gm)	$m = m_t + \Delta m$ m_t (gm)	ω_D cps	$\omega_D \sqrt{m}$ cps gm ^{1/2}	output g's	A	$\eta =$ $1/\sqrt{A^2 - 1}$
0.020	0.5	2.15	0.4	2.55	115	184	9.0	18.0	.056
	0.5	4.0		4.40	87	182	10.0	20.0	.050
	0.5	6.9		7.30	61	164	8.5	17.0	.059
.035	0.5	2.2	0.4	2.6	118	190	5.0	10.0	.100
	0.5	4.0		4.4	86	130	4.5	9.0	.111
	0.5	6.9		7.3	62	168	5.0	10.0	.100
.062	.5	2.15		2.65	129	210	4.0	6.9	.146
	.5	4.0	0.5	4.5	97	205	3.5	7.0	.144
	.5	6.9		7.4	70	190	3.0	6.0	.169
.078	.5	2.15		2.65	124	203	2.8	5.6	.183
	.5	2.15	0.5	2.65	130	212	3.5	7.0	.144
	.5	4.0		4.50	94	199	2.6	5.2	.197
	.5	6.9		7.40	67	182	2.2	4.4	.234
.125	.5	2.15	0.6	2.75	135	224	5.0	10.0	.100
	.5	4.0		4.6	97	208	4.0	8.0	.126
	.5	6.9		7.5	70	192	3.2	6.4	.158

TABLE VII

VALUES OF $(\omega_D/\omega_1) (1+\psi_e)^{1/2} (1+\eta^2)^{1/4}$ FOR OPTIMAL TUNING OF CANTILEVER BEAM
WITH 1:1 DISTRIBUTED TUNED DAMPERS (TYPE A) UNDER SHAKER EXCITATION

TEST BEAM THICKNESS $h = 0.25$ inches

$\eta = 0.175, (1+\eta^2)^{1/4} \approx 1, \text{Temp} \approx 80^\circ\text{F}$

$\psi = \psi_e, l_D = 1.125 \text{ in.}, b = 0.5 \text{ in.}, h_D = 0.02 \text{ in.}$

τ ins	m_t gm	Δm gm	$m = m_t + \Delta m$ gm	ω_D cps (Fig 15)	L ins	ω_1 cps	ψ	$\frac{\omega_D \sqrt{1+\psi}}{\omega_1}$	input g's	Low Freq peak		High Freq peak		A	$n_s = \frac{1}{\sqrt{A^2-1}}$
										freq cps	output g's	freq cps	output g's		
.020	2.1	.4	2.5	106	7.30	119	.221	.985	0.5	89	7.60	139	7.60	15.2	.066
	7.5		7.9	60	8.92	85	.650	.910	0.5	--	9.40	--	9.40	19.8	.050
	4.7		5.1	74	8.78	87	.605	1.01	0.5	54	7.80	109	7.80	15.6	.064
.062	7.5	.5	8.0	62	9.05	83	.640	0.96	0.5	47	4.50	109	4.40	9.0	.112
	*4.7		5.2	77	8.60	90	.412	1.02	1.0	60	7.20	126	7.30	7.3	.139
	*4.7		5.2	77	8.50	90	.412	1.02	0.5	59	3.60	128	3.60	7.2	.140
	7.5		8.0	62	9.00	83	.640	0.96	1.0	48	8.70	116	8.80	8.8	.114
.078	7.5	.5	8.0	66	8.96	84	.650	1.01	1.0	49	8.50	114	8.50	8.5	.118
	7.5		8.0	66	8.90	85	.650	1.00	0.5	49	4.30	112	4.40	8.8	.114
	2.1		2.6	117	6.95	131	.236	1.00	0.5	101	4.50	176	4.50	9.0	.112
	2.1		2.6	117	6.87	133	.236	0.98	0.5	102	4.10	170	4.70	8.6	.117
	2.1		2.6	117	6.95	131	.236	1.00	1.0	104	8.60	174	8.60	8.6	.117
.125	7.5	.6	8.1	77	8.40	94	.700	1.07	0.5	56	5.10	124	5.10	10.2	.100
	2.1		2.7	134	6.94	131	.233	1.12	0.5	100	5.00	175	5.00	10.2	.100

* Additional data not included in [9].

TABLE VIII

LOSS FACTOR AND NATURAL FREQUENCIES OF TYPE B CANTILEVER DAMPERS

 $\ell_D = 1.125$ ins., $b = 0.50$ ins., $h_D = 0.020$ ins.

τ ins	input g's	m_t gm	$\Delta m =$ $0.338 + 2.78\tau$ gm	$m_t + \Delta m$ gm	ω_D cps	$\omega_D \sqrt{m}$ cps gm ^{1/2}	Temp °F	Output g's	A	$\eta =$ $\frac{1}{\sqrt{A^2 - 1}}$
.062	0.40	37	0.51	37.5	28	171	77	1.32	3.30	.318
	0.40	37	0.51	37.5	29	178	82	1.48	3.71	.280
	0.39	37	0.51	37.5	26	159	82	1.30	3.34	.316
.076	0.51	37	0.56	37.6	33	202	80	1.32	2.60	.416
	0.40	37	0.56	37.6	29	178	82	1.14	2.85	.375
	0.40	37	0.56	37.6	29	178	81	1.22	3.05	.350
	0.40	37	0.56	37.6	30	184	81	1.15	2.86	.375
	0.40	37	0.56	37.6	30	184	80	1.22	3.05	.350
	0.39	37	0.56	37.6	30	184	79	1.18	3.02	.350
	0.40	37	0.56	37.6	30	184	79	1.27	3.18	.332
	0.40	37	0.56	37.6	30	184	78	1.32	3.30	.318
	0.40	37	0.56	37.6	27	165	82	1.24	3.10	.342
	0.40	37	0.56	37.6	29	178	82	1.25	3.12	.337
	0.40	37	0.56	37.6	30	184	82	1.24	3.10	.342
.125	0.39	37	0.69	37.7	32	196	82	1.65	4.25	.243
	0.40	37	0.69	37.7	32	196	82	1.90	4.75	.216
	0.40	37	0.69	37.7	32	196	78	2.32	5.80	.176
	0.40	37	0.69	37.7	30	184	78	-	-	-
	0.39	37	0.69	37.7	28	172	78	1.33	3.32	.316
	0.39	37	0.69	37.7	30	184	81	1.80	4.50	.223
	0.39	37	0.69	37.7	32	196	78	2.21	5.55	.183

TABLE IX

VALUES OF $(\omega_D/\omega_1) (1+\psi_e)^{1/2} (1+\eta^2)^{1/4}$ FOR OPTIMAL TUNING OF CANTILEVER BEAM WITH

TYPE B TUNED DAMPER AT FREE END, UNDER SHAKER EXCITATION (FROM [4])

TEST BEAM THICKNESS = 0.125 ins.

 $\psi_e = 4\psi$, $l_D = 1.125$ ins., $h_D = 0.02$ ins., input = 0.5 g's

τ ins	ψ	m_t gm	m gm	$m = m_t + \Delta m$ gm	$\omega_D \sqrt{m}$ (Fig 16)	ω_D cps	L ins	ω_1 cps	$\frac{\omega_D}{\omega_1} \sqrt{1+\psi_e}$	Temp °F	Low Freq peak freq output cps g's	High Freq peak freq output cps g's	$\eta_s =$ $\frac{1.56}{\sqrt{A^2-1}}$
0.062	.257	15.75	.51	16.2	176	44	5.50	72	.89	77	35 2.49	82 2.70	.306
	.283	18.0	.51	18.5	176	41	5.80	64	.95	78	33 2.49	76 2.65	.310
	.314	20.3	.51	20.7	176	39	5.89	64	.93	77	31 2.40	73 2.62	.317
($\tau = 0.325$)													
0.078	.063	2.25	.56	2.8	186	111	3.08	184	.71	81	112 2.36	195 2.43	.334
	.102	4.50	.56	5.0	186	84	3.70	140	.74	81	81 2.27	152 2.40	.342
	.138	6.75	.56	7.2	186	69	4.20	113	.79	82	63 2.05	130 2.27	.372
	.170	9.00	.56	9.5	186	60	4.59	98	.83	82	53 2.30	114 2.21	.356
	.202	11.25	.56	11.8	186	54	4.90	88	.86	83	45 2.48	102 2.05	.355
($\tau = 0.40$)													
	.328	20.25	.56	20.7	186	41	5.54	70	.93	83	32 1.90	79 2.08	.403
0.125	.014	0	.69	0.69	220	265	2.39	285	.97	82	180 3.8	286 3.8	.205
	.060	2.05	.69	2.74	220	132	3.35	164	.92	80	101 3.9	182 3.5	.211
	.101	4.5	.69	5.19	220	97	3.80	134	.88	80	76 3.4	152 4.0	.211
	.136	6.75	.69	7.3	220	82	4.26	110	.94	80	62 3.8	130 3.8	.205
	.170	9.00	.69	9.6	220	71	4.60	96	.97	82	52 3.8	117 3.6	.211
	.201	11.25	.69	11.8	220	64	4.89	88	.99	82	47 3.7	107 3.6	.211
	.234	13.5	.69	14.1	220	59	5.08	81	1.03	82	41 3.4	99 3.6	.226
	.265	15.75	.69	16.4	220	54	5.30	76	1.05	82	38 3.3	92 3.5	.232
	.292	18.0	.69	18.6	220	51	5.56	70	1.09	82	35 3.2	86 3.3	.244
($\tau = 0.22$)													
	.322	20.25	.69	20.9	220	48	5.69	68	1.09	82	32 3.2	80 3.2	.251

TABLE X

LOSS FACTORS AND NATURAL FREQUENCIES OF TYPE C TUNED DAMPERS

 $h_D = 0.063 \text{ ins.}, b = 1.0 \text{ ins.}$

ℓ_D ins	τ ins	m_t gm	Δm gm	$m = m_t + \Delta m$ gm	ω_D cps	$\omega_D m^{1/2} \ell_D^{3/2}$	Temp °F	input g's	output g's	A	$\eta = 1/\sqrt{\lambda^2 - 1}$
3.7	0.020	7.6	2.9	10.5	71	1635	78	0.5	9.3	18.6	.054
					71	1635	79		9.4	18.8	.053
					71	1635	82		9.5	19.0	.053
					72	1660	82		9.5	19.0	.053
	0.035	7.6	3.2	10.8	71	1660	79	0.5	7.82	15.64	.064
					71	1660	81		8.20	16.4	.061
	0.062	7.6	3.7	11.3	70	1680	82	0.5	5.4	10.8	.093
					71	1700	79		5.2	10.4	.096
					73	1748	81		5.95	11.9	.084
	0.078	7.6	4.3	11.9	77	1880	81	0.5	4.25	8.5	.118
					72	1740	81		3.90	7.8	.129
					74	1810	79		3.7	7.4	.137
					72	1740	82		4.1	8.2	.123
	0.125	7.6	5.0	12.6	83	2060	79	0.5	2.25	4.5	.226
					80	2020	80		2.28	4.56	.225
					79	1990	80		2.3	4.6	.224
					80	2020	81		2.28	4.56	.225
					79	1990	81		2.35	4.7	.218
					78	1960	82		2.4	4.8	.214

TABLE X (CONT'D)

ℓ_D ins	τ ins	m_t gm	Δm gm	$m =$ $m_t + \Delta m$ gm	ω_D cps	$\omega_D m^{1/2}$ $\ell_D^{3/2}$	Temp °F	input g's	output g's	A	$\eta =$ $1/\sqrt{\lambda^2 - 1}$
2.2	0.020	36.8	1.7	38.5	83	1670	78	0.5	11.2	22.4	.044
					83	1670	78	0.3	9.7	32.3	.031
	0.078	37.0	2.6	39.6	92	1880	83	0.5	4.4	8.8	.114
		41.8		44.4	80	1730	76	1.0	7.4	7.4	.136
		41.8		44.4	82	1780	76	0.5	3.5	7.0	.144
	0.125	45.8	3.0	48.8	91	2040	76	1.0	5.2	5.2	.196
					91	2040	76	0.5	2.5	5.0	.203
					81	1840	79	0.5	2.85	5.7	.178
2.7	0.020	28.0	2.1	30.1	72	1750	82	0.5	10.9	21.8	.046
		28.0		30.1	72	1750	82	0.3	7.7	25.7	.039
		21.2		23.3	78	1670	84	0.5	11.5	23.0	.044
		21.2		23.3	78	1670	84	0.3	9.9	33.0	.030
	0.078	28.0	3.1	31.1	72	1780	80	0.5	3.72	7.44	.138
					73	1800	76	1.0	6.65	6.65	.152

TABLE XI

VALUES OF $(\omega_D/\omega_1)(1+\psi_e)^{1/2}(1+\eta^2)^{1/4}$ FOR CLAMPED-CLAMPED BEAM WITH TUNED DAMPER.

AT CENTER UNDER SHAKER EXCITATION (FROM [J])

TEST BEAM THICKNESS = 0.217 inches

 $\psi_e = 2.5\psi$, $\omega_1 = 90$ cpsTemp = 80°F ($\pm 3^\circ\text{F}$)

l_D ins	τ ins	m_t gm	Δm gm	m gm	$\omega_D l_D^{3/2} \sqrt{m}$ Fig. 26	ω_D cps	$\psi =$ m/396	$\frac{\omega_D \sqrt{1+\psi_e}}{\omega_1}$ $(1+\eta^2)^{1/4}$	input g's	Low Freq peak freq output cps g's		High Freq peak freq output cps g's	
										freq cps	output g's	freq cps	output g's
2.7	0.020	14.5	2.1	16.6	1660	92	.042	1.06	0.3	73	8.0	107	7.7
		17.7	2.1	19.8		84	.052	1.00		72	4.0	106	3.9
	0.062	17.9	2.8	20.7	1740	87		1.03		71		107	4.0
		16.5	3.0	19.5	1800	92	.049	1.08	0.5	70	5.2		5.2
	0.125	26.1	3.7	29.8	2000	82	.075	1.00		67	3.2	108	3.3
		23.8		27.5		86	.070	1.04	0.75			107	5.2
		20.9		24.6		91	.062	1.09	2.0	68	15.0	106	15.5
		23.2		26.9		87	.068	1.05	1.0			107	7.0
		25.5		29.0		83	.073	1.00	0.75	67	5.0	108	5.3
		28.2		31.9		80	.081	0.98	0.3	65	1.8	107	1.9
3.7	0	2.4	2.5	4.9	1660	106	.012	1.20	0.25	80	33	103	33
									0.5		54	100	56
	0.020	2.8	2.9	5.7		98	.014	1.11		77	18.2	101	19.2
									0.3			100	10.8
	0.035	2.4	3.2	5.6					0.5		16.5	102	13.5
									1.0	76	27.5	101	27.7
	0.062	3.0	3.8	6.8	1740	95	.017	1.07	0.3	75	5.2	102	5.1
									0.75	76	12.5	103	12.5
									1.0	75	16	101	76
									1.5		23	103	24.5
0.073		3.1	4.1	7.2	1800	91	.018	1.03	0.5	74	6.1		5.9
									1.0		11.8	101	11.8
	0.125	4.3	5.1	9.4	2000	92	.024	1.05	0.5	73	3.5	103	3.5
									1.0	71	6.7	101	6.8

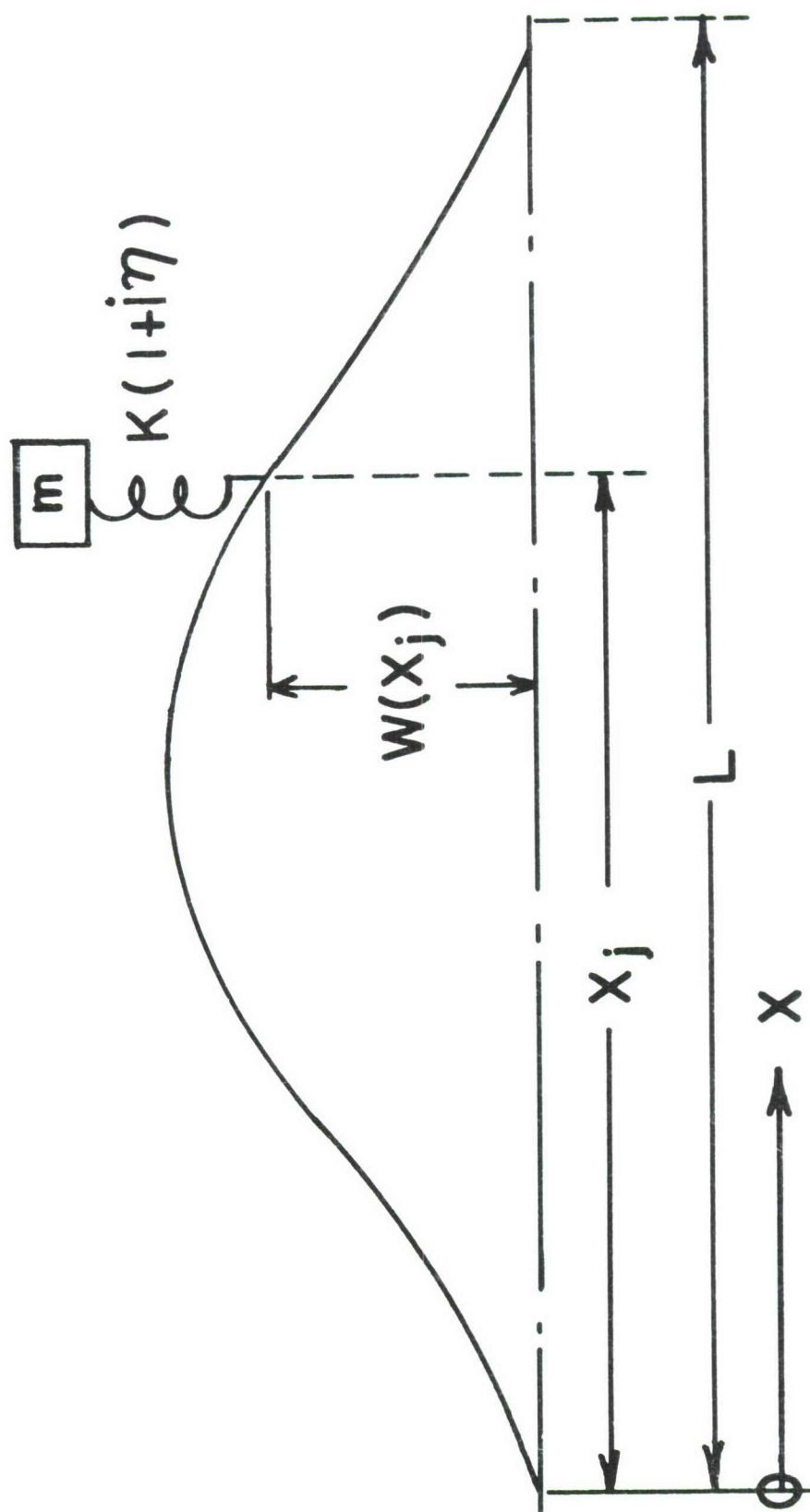


Figure 1. Idealized Beam - Damper System

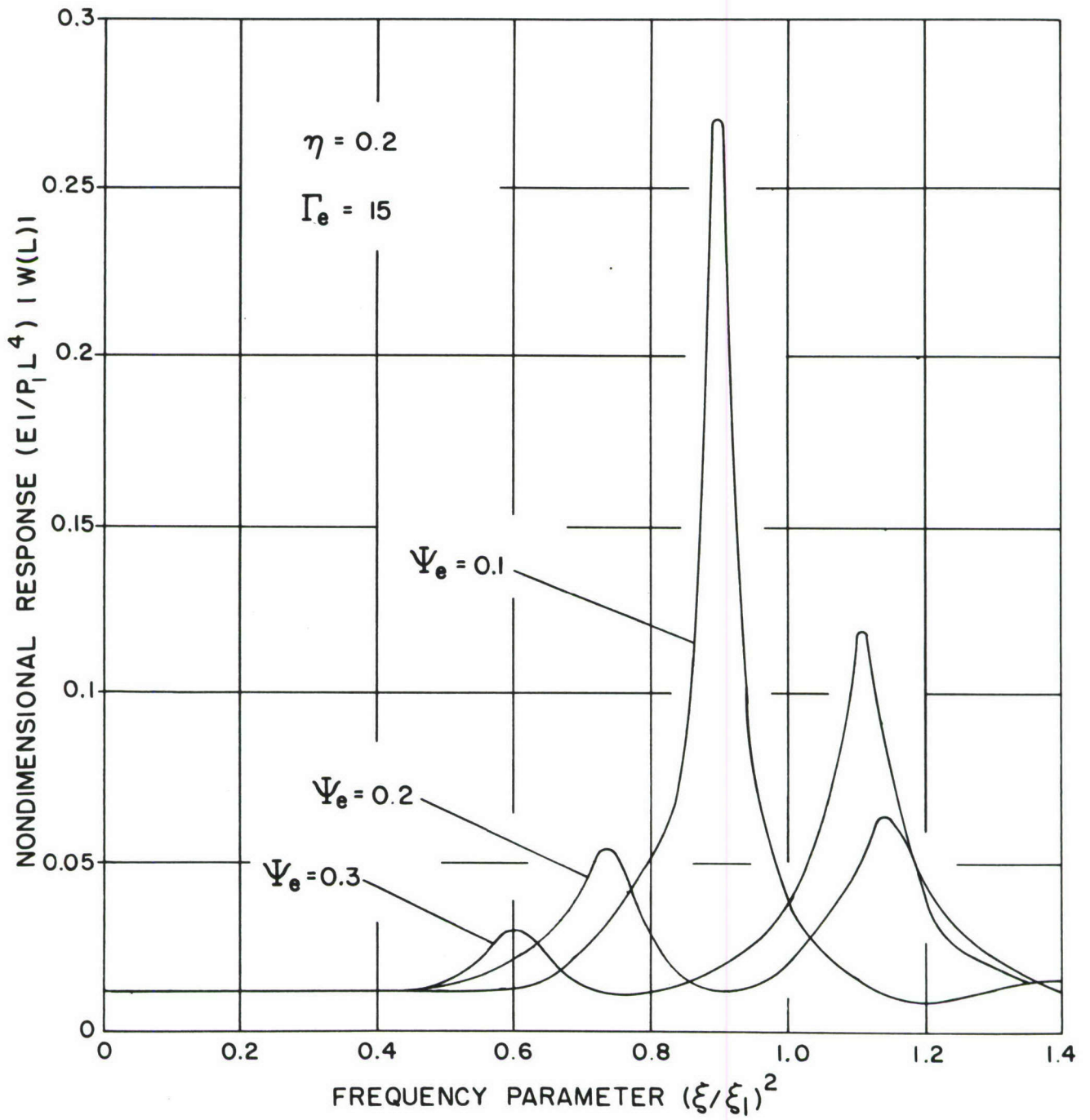


Figure 2. Typical Response Spectra for $\eta = 0.2$

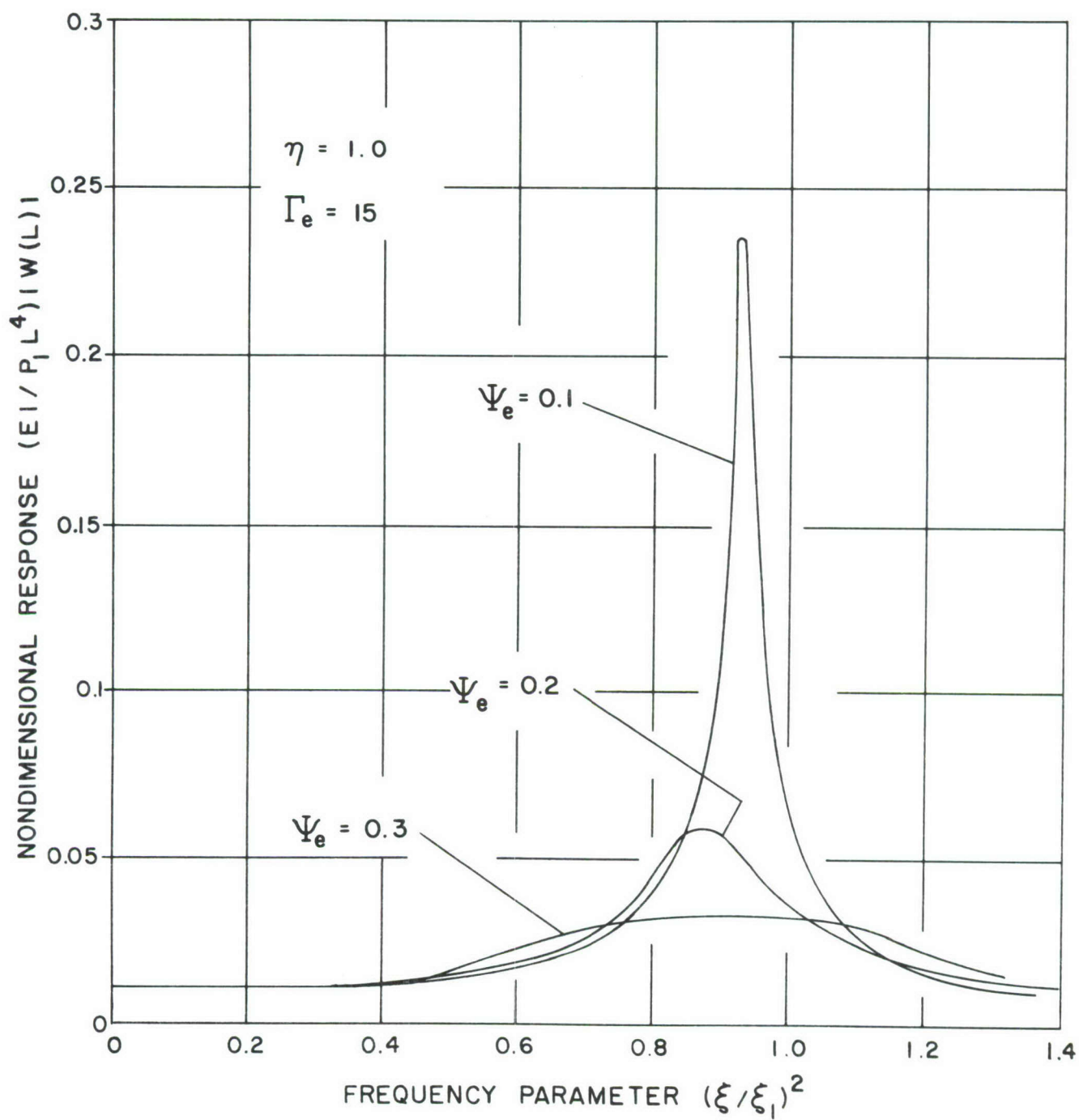


Figure 3. Typical Response Spectra for $\eta = 1.0$

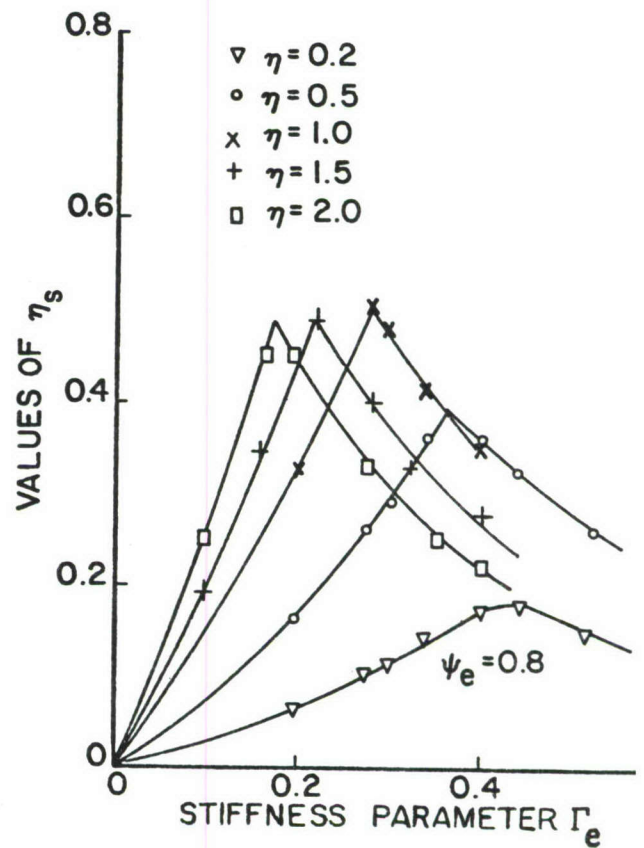
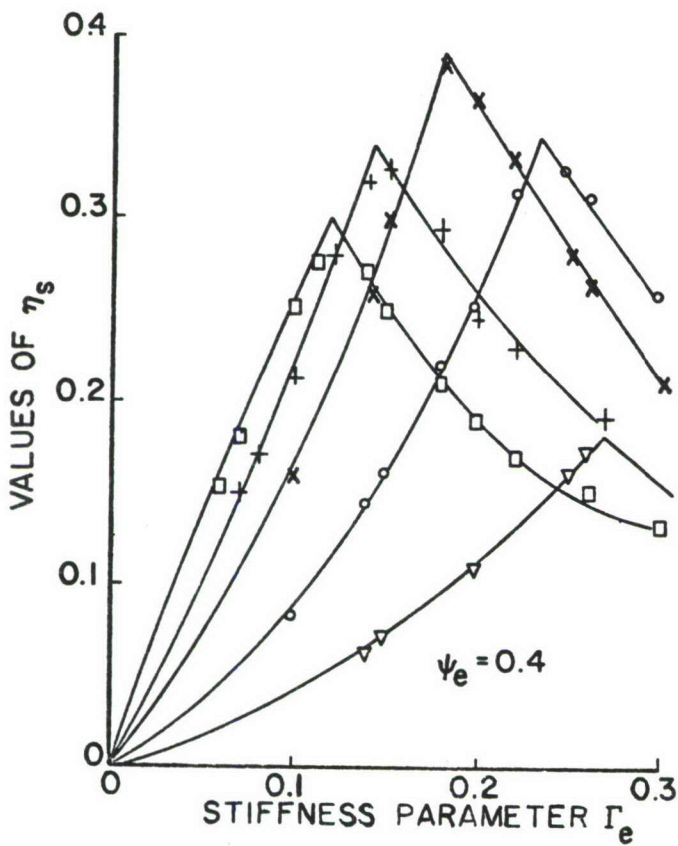
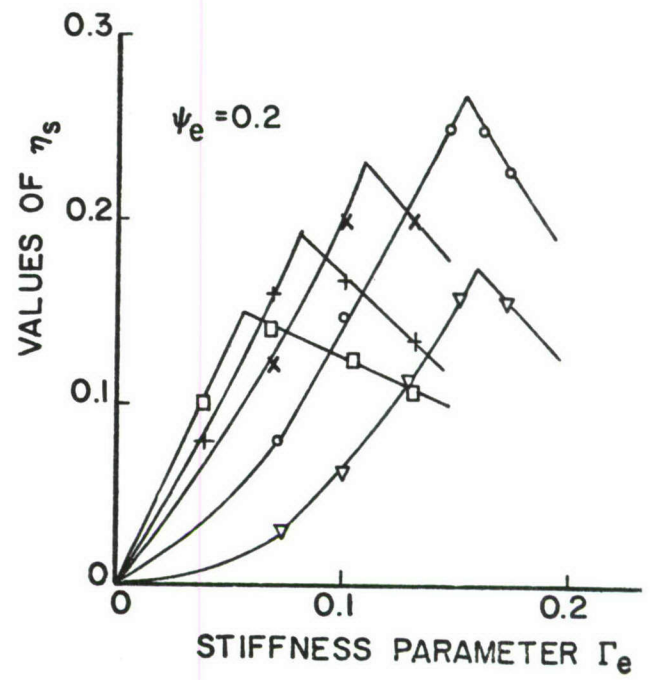
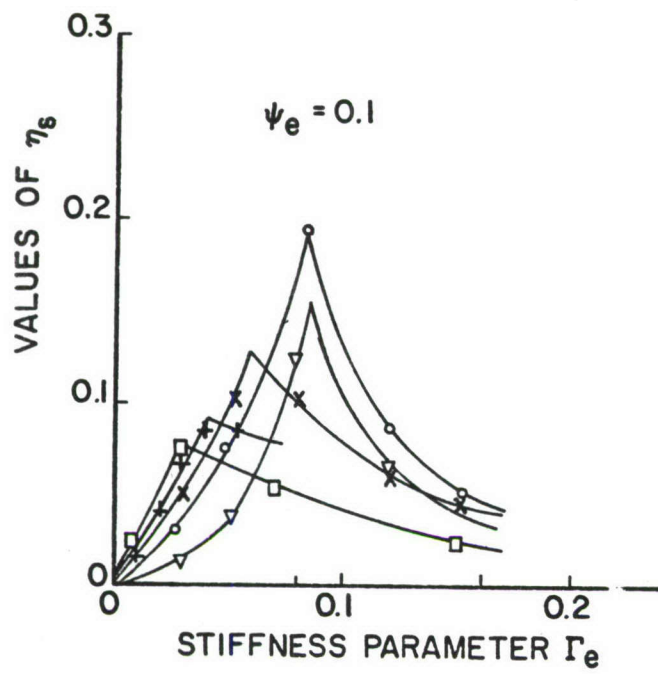


Figure 4. Graphs of Effective Loss Factor η_s Against Stiffness Parameter Γ_e (Force Excitation)

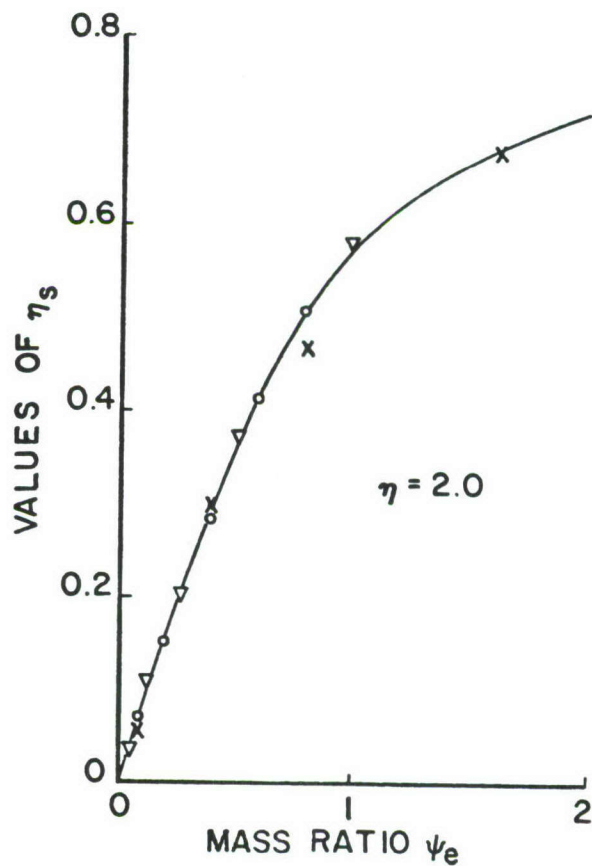
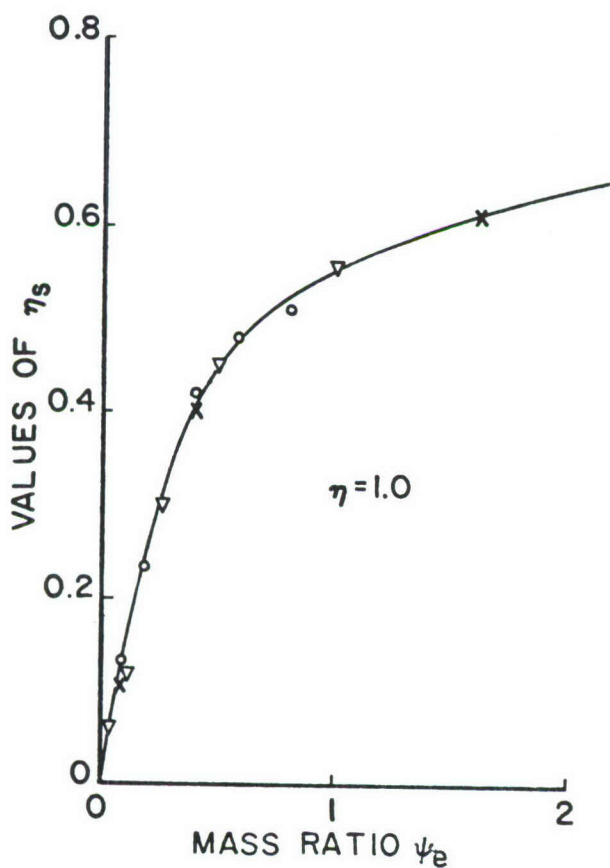
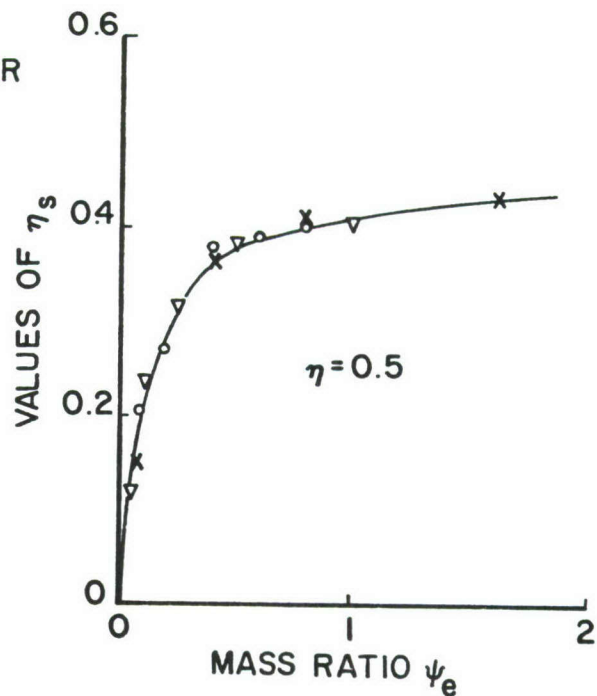
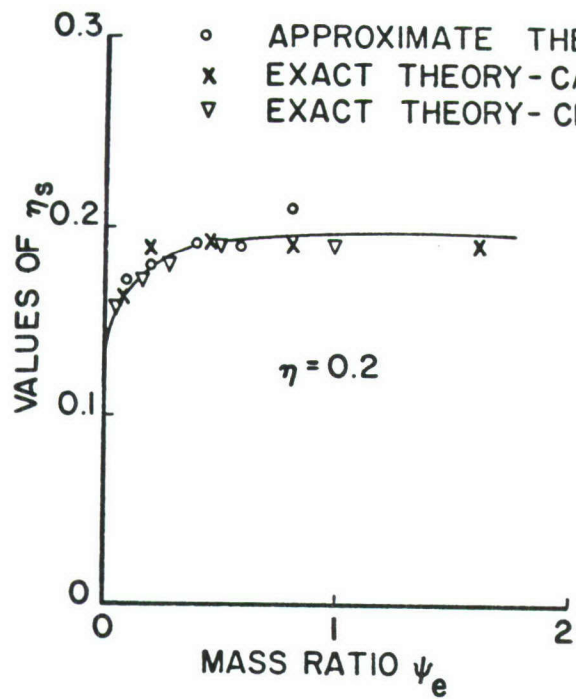


Figure 5. Graphs of Optimum Effective Loss Factor Against Effective Mass Ratio ψ_e

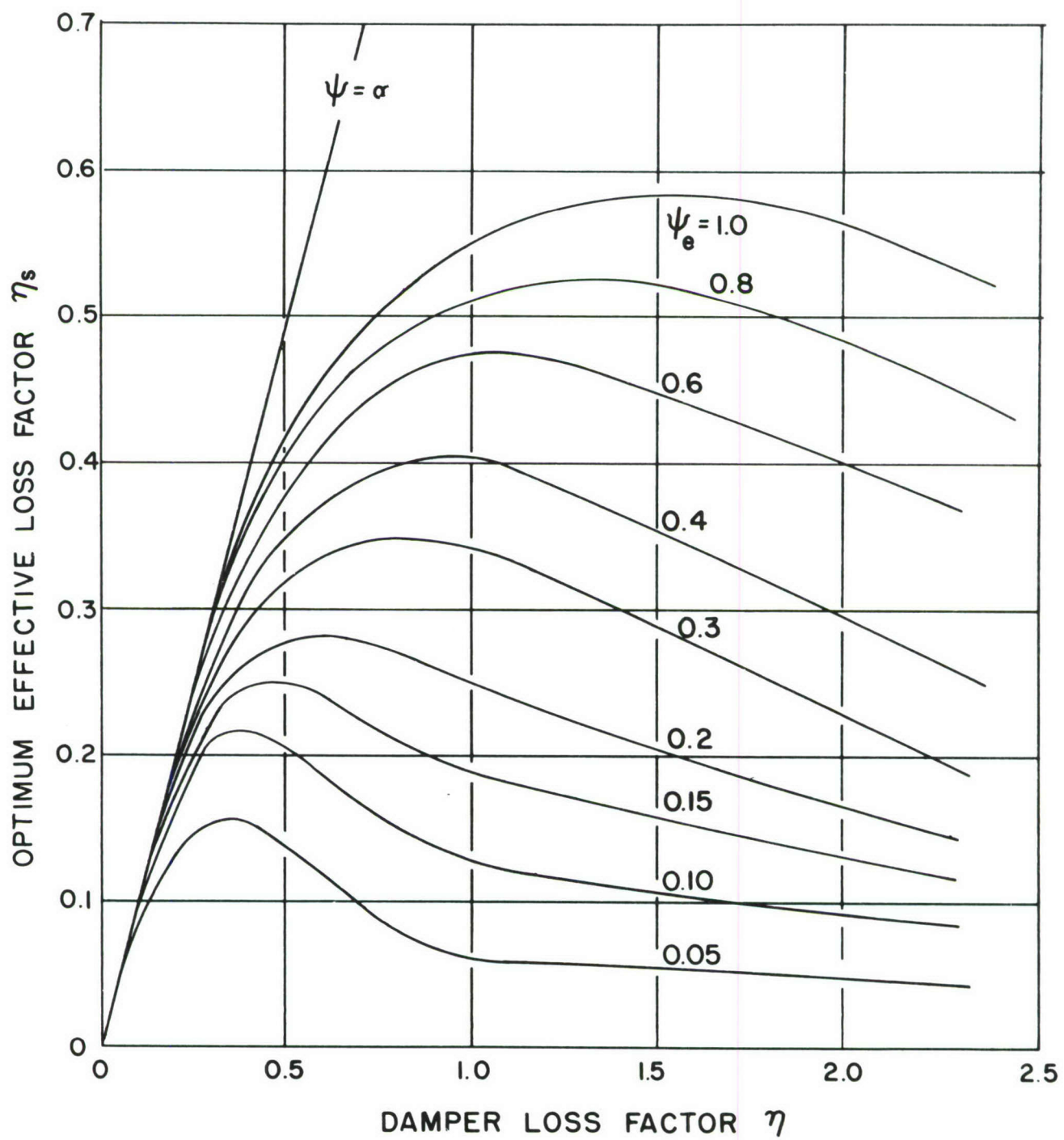
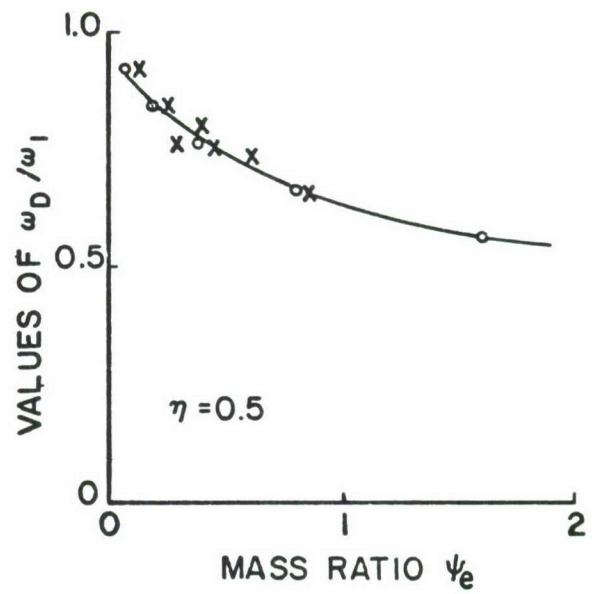
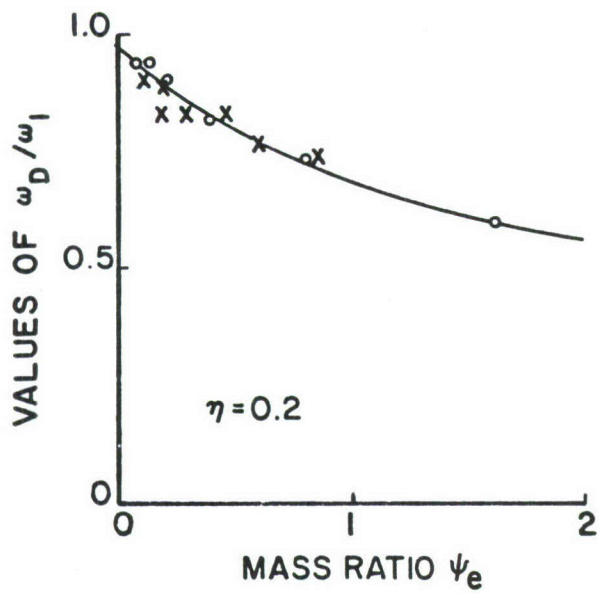


Figure 6. Graphs of Optimum Effective Loss Factor Against Damper Loss Factor



○ APPROX. THEORY
 x EXACT THEORY
 (CANTILEVER)

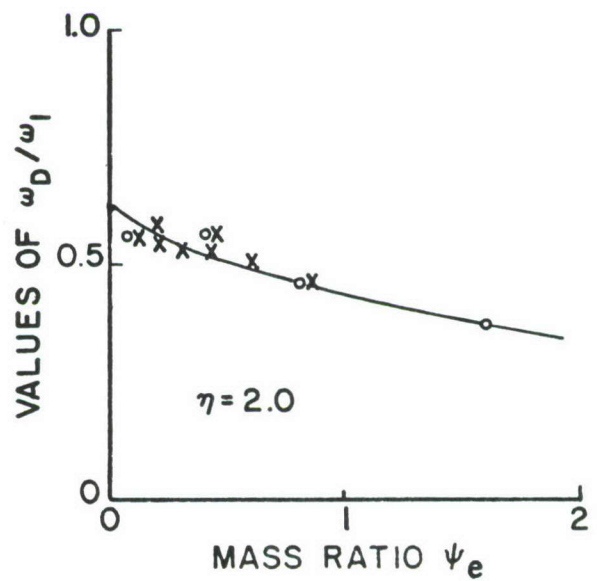
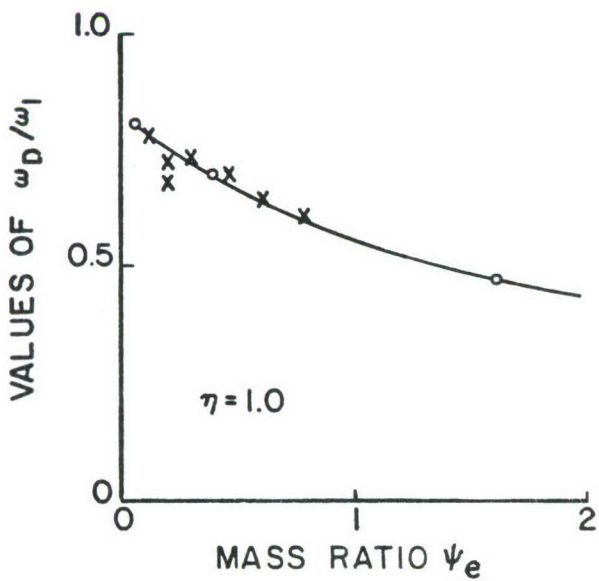


Figure 7. Graphs of ω_D/ω_1 Against ψ_e for Optimal Tuning
 (Force EXcitation)

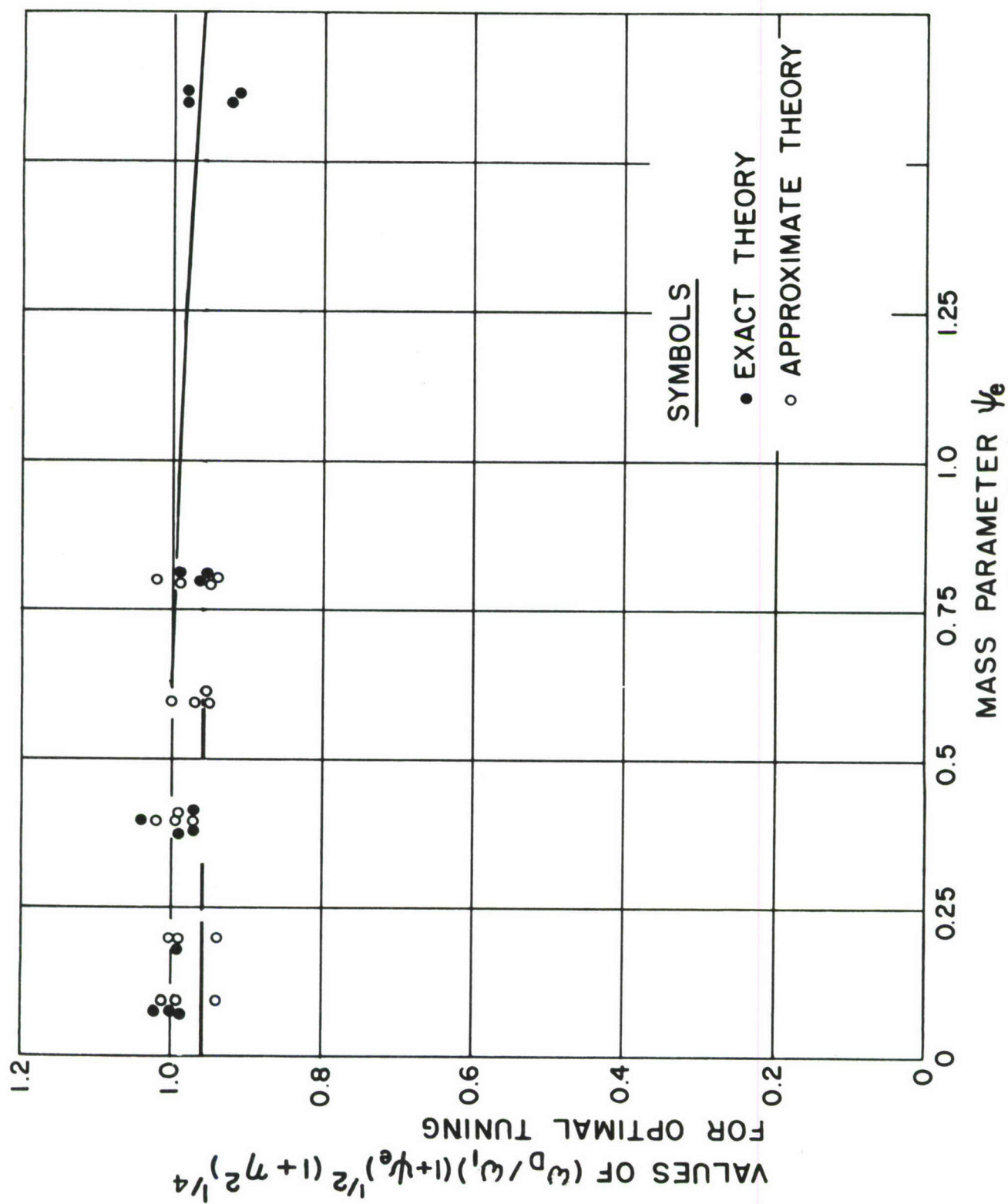


Figure 8. Graphs of $(\omega_D / \omega_1)(1 + \psi_e)^{1/2} (1 + \eta^2)^{1/4}$ against ψ_e for Optimal Tuning (Force Excitation)

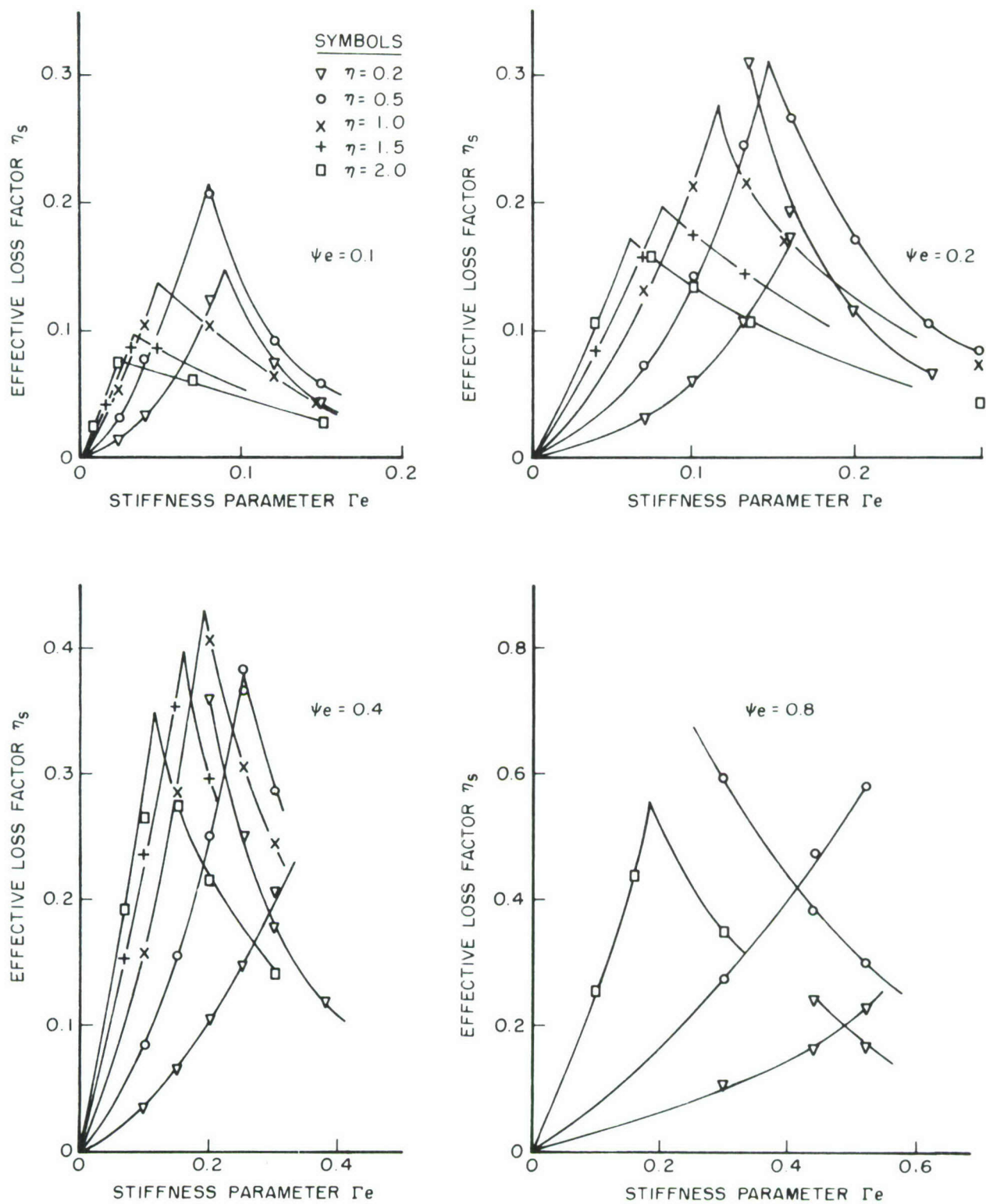


Figure 9. Graphs of Effective Loss Factor η_s Against Stiffness Parameter Γ_e (Shaker Excitation)

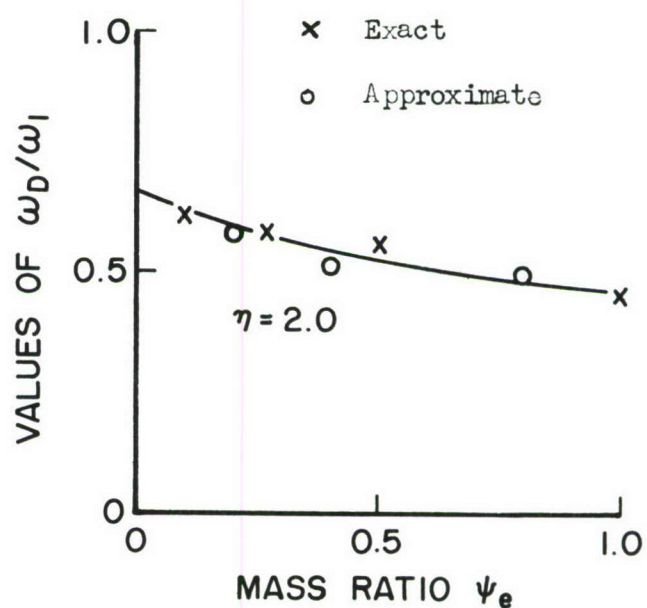
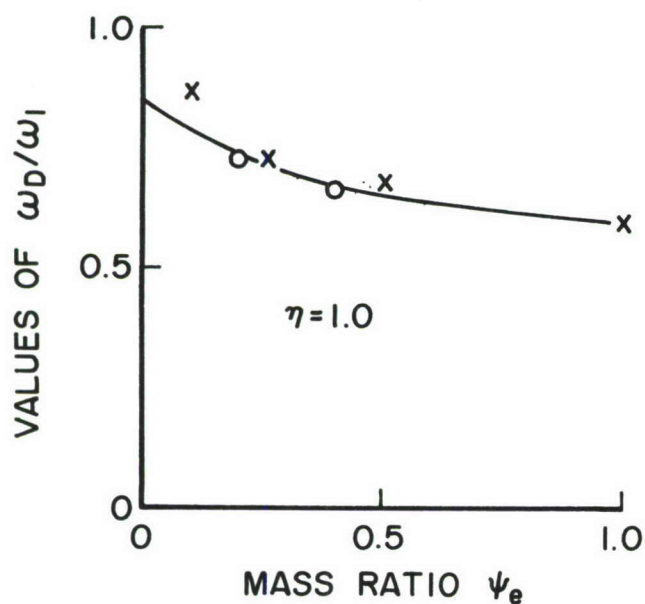
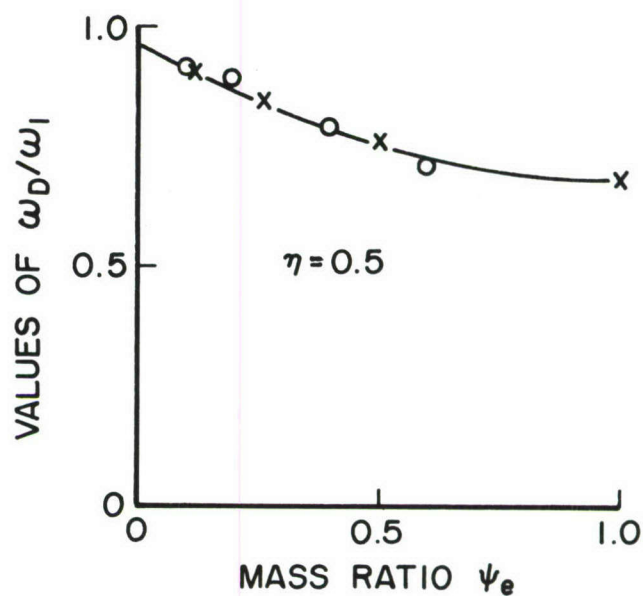
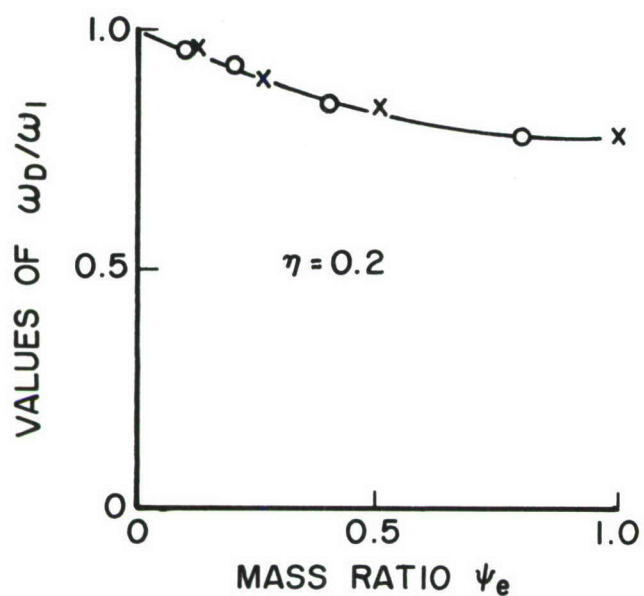


Figure 10. Graphs of ω_D/ω_1 Against ψ_e for Optimal Tuning

(Shaker Excitation)

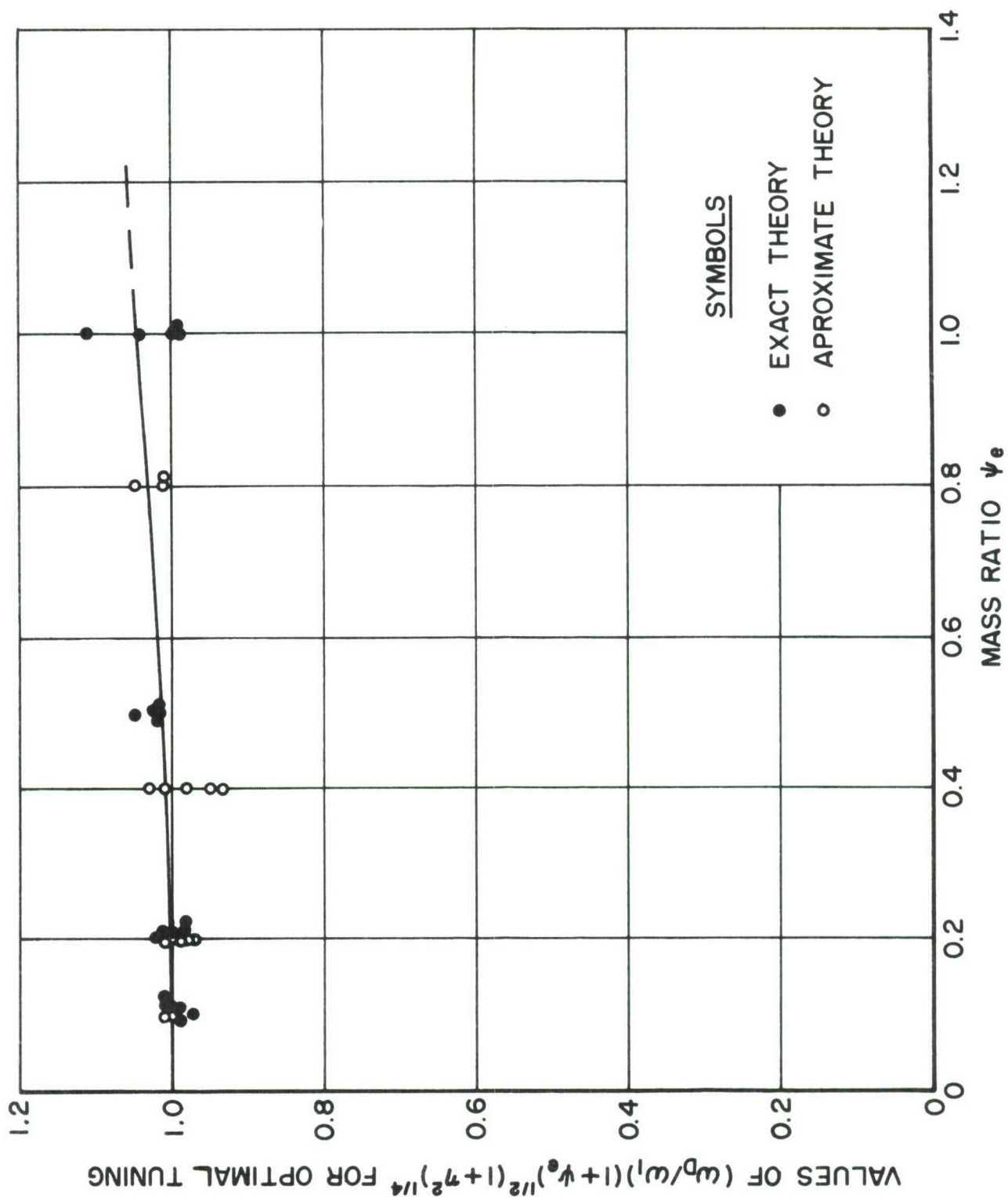


Figure 11. Graph of $(\omega_D/\omega_1)((1+\psi_e)^{1/2}(1+\eta^2)^{1/4})$ against ψ_e for Optimal Tuning (Shaker Excitation)

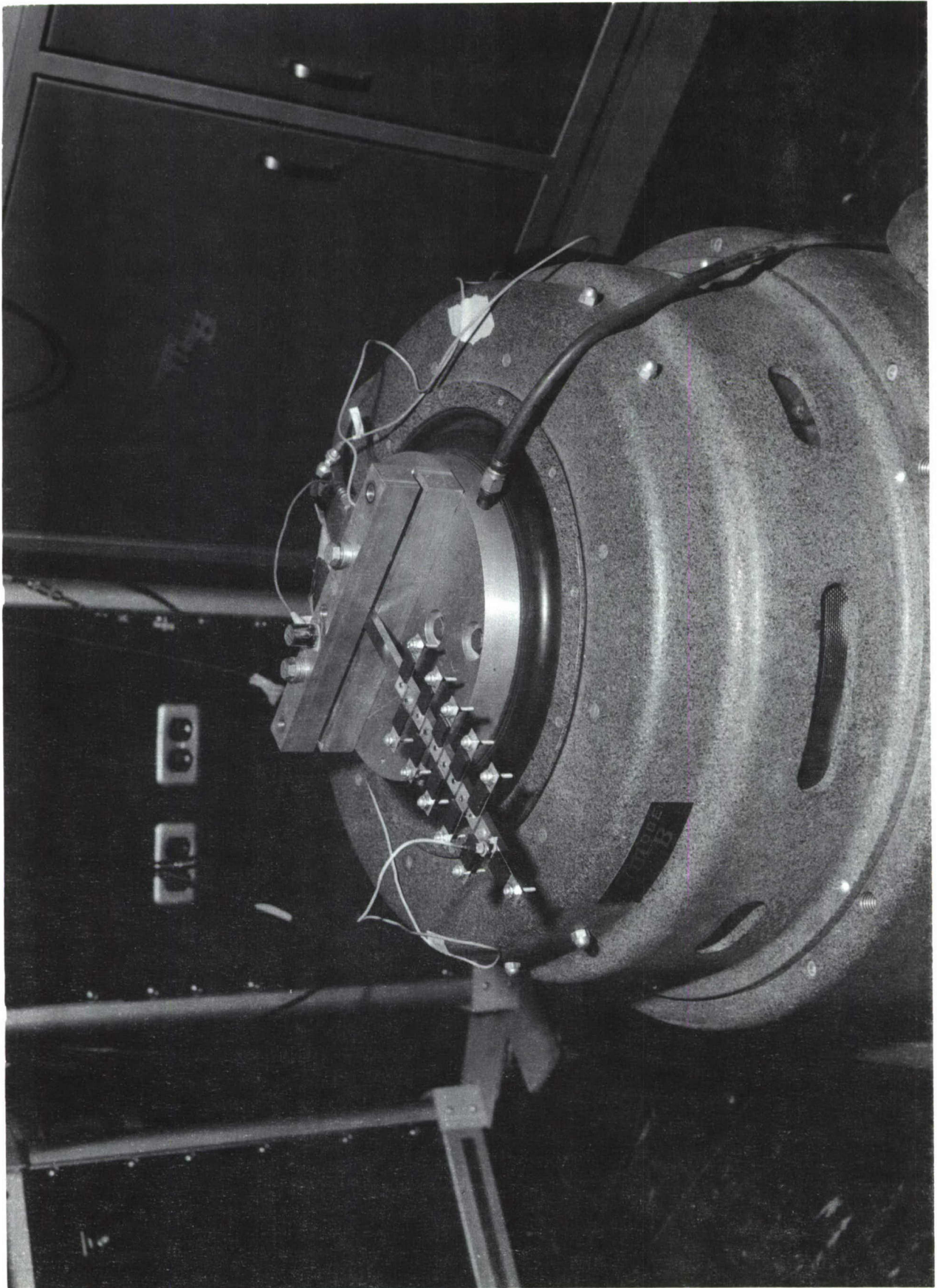


Figure 12. Photograph of Beam-Damper System

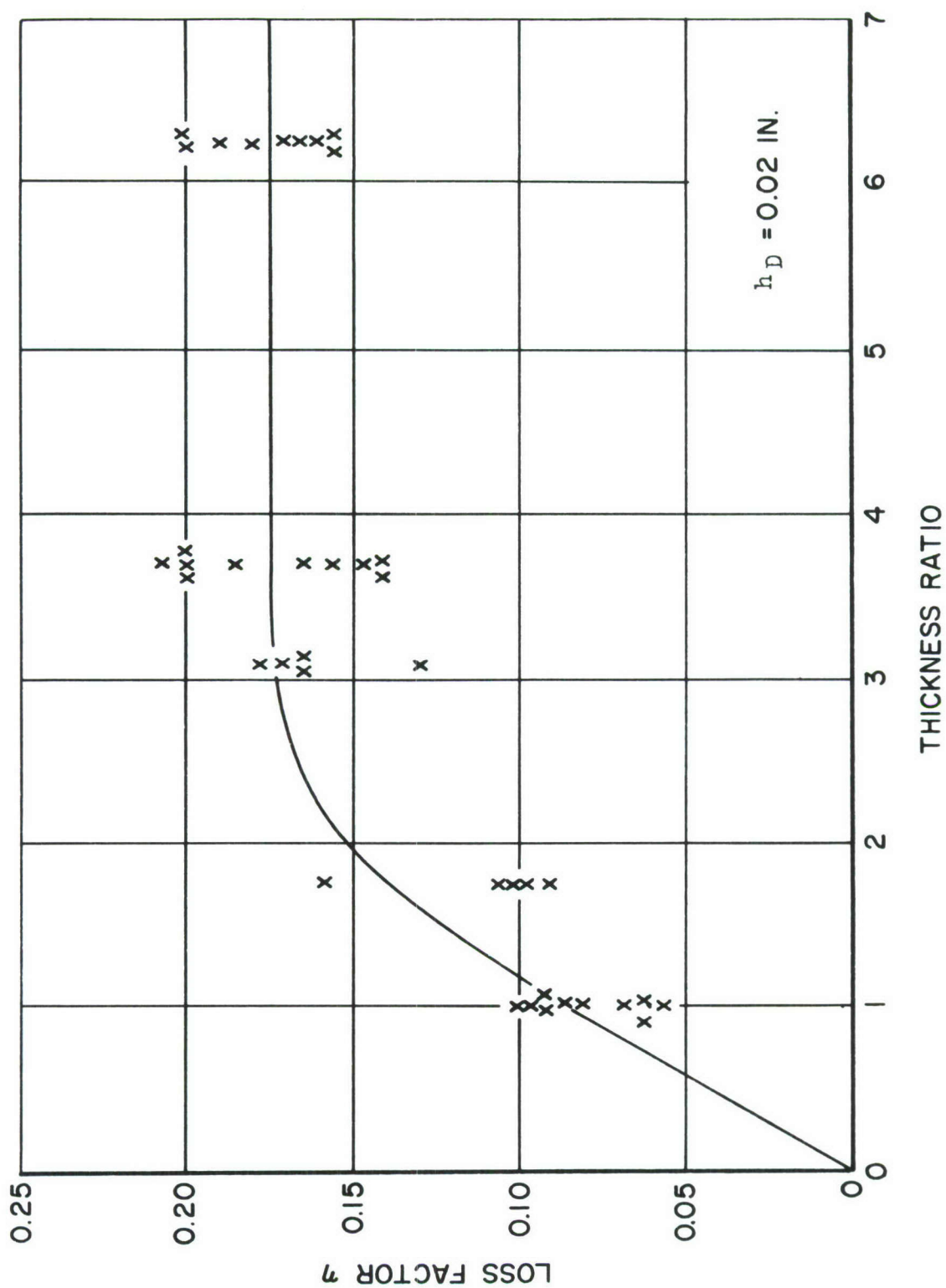


Figure 13. Graph of Dumper Loss Factor η against Thickness Ratio τ/η_D for Cantilever Dampers Used as Distribution on Cantilever Beams

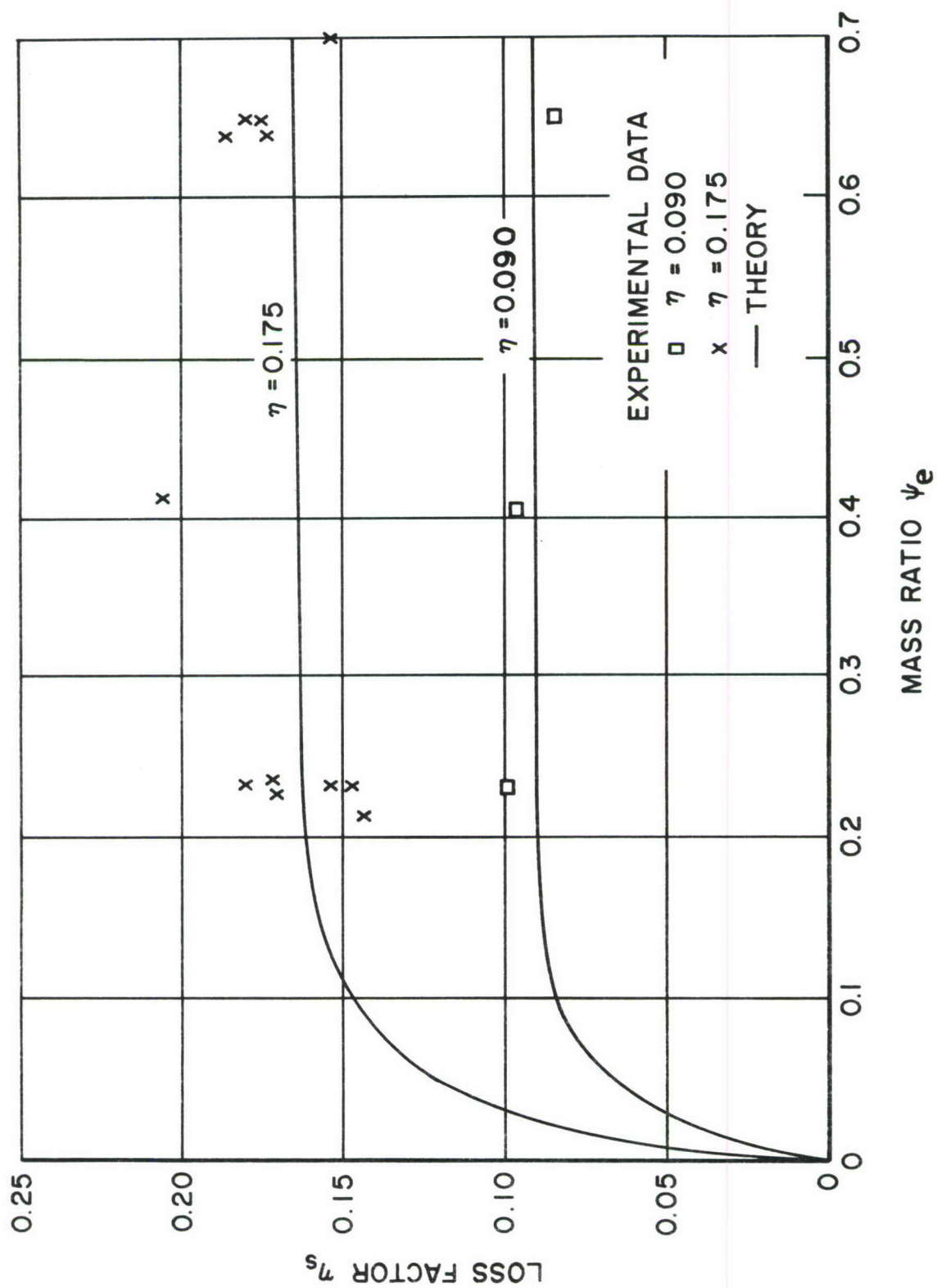
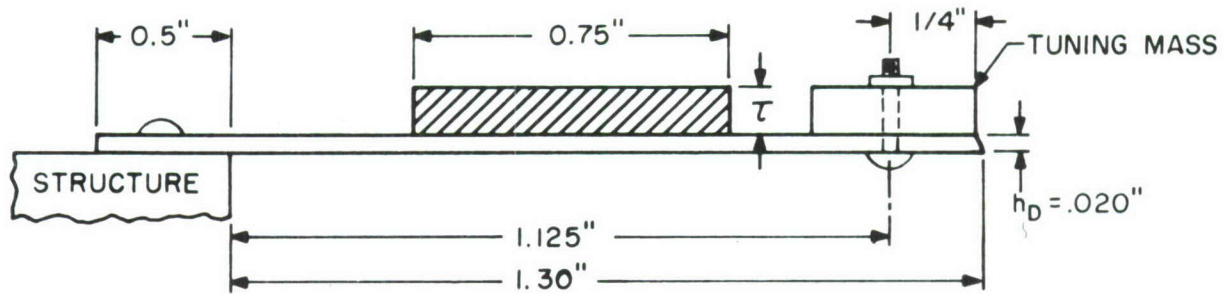
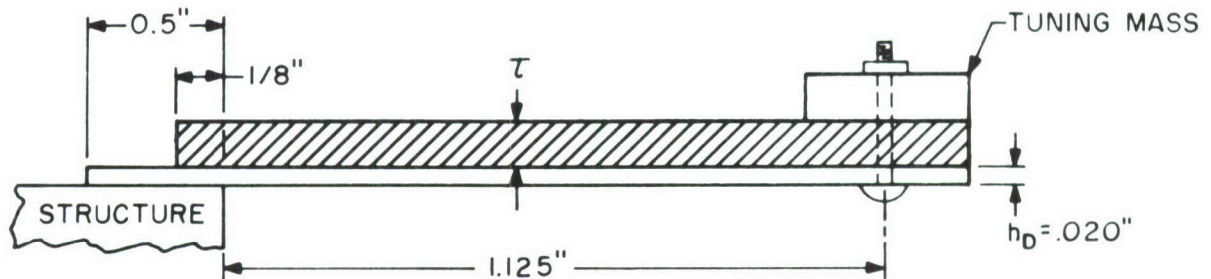


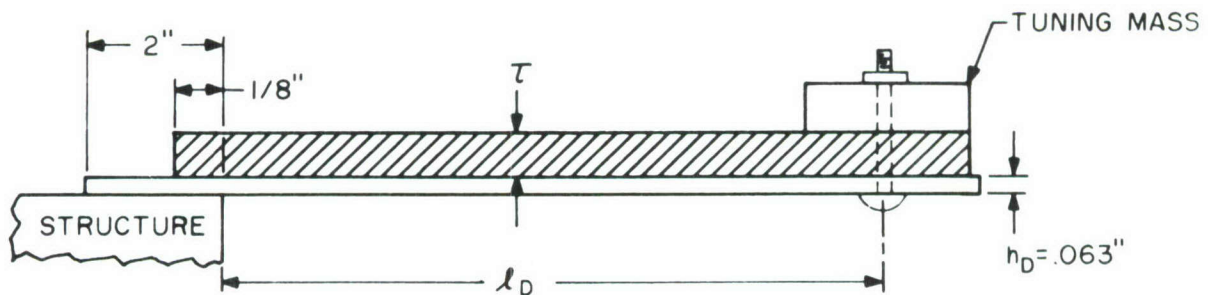
Figure 14. Experimental Values of η_s Plotted Against ψ_e for $\eta = 0.175$ and 0.09



(a) LOW LOSS FACTOR UNITS USED ON CANTILEVER BEAM (DISTRIBUTED DAMPERS)



(b) HIGH LOSS FACTOR UNITS USED ON CANTILEVER BEAM (SINGLE DAMPER AT FREE END)



(c) DAMPER UNITS USED ON CLAMPED-CLAMPED BEAM

Figure 15. Sketches of Tuned Dampers Used in Experimental Investigations

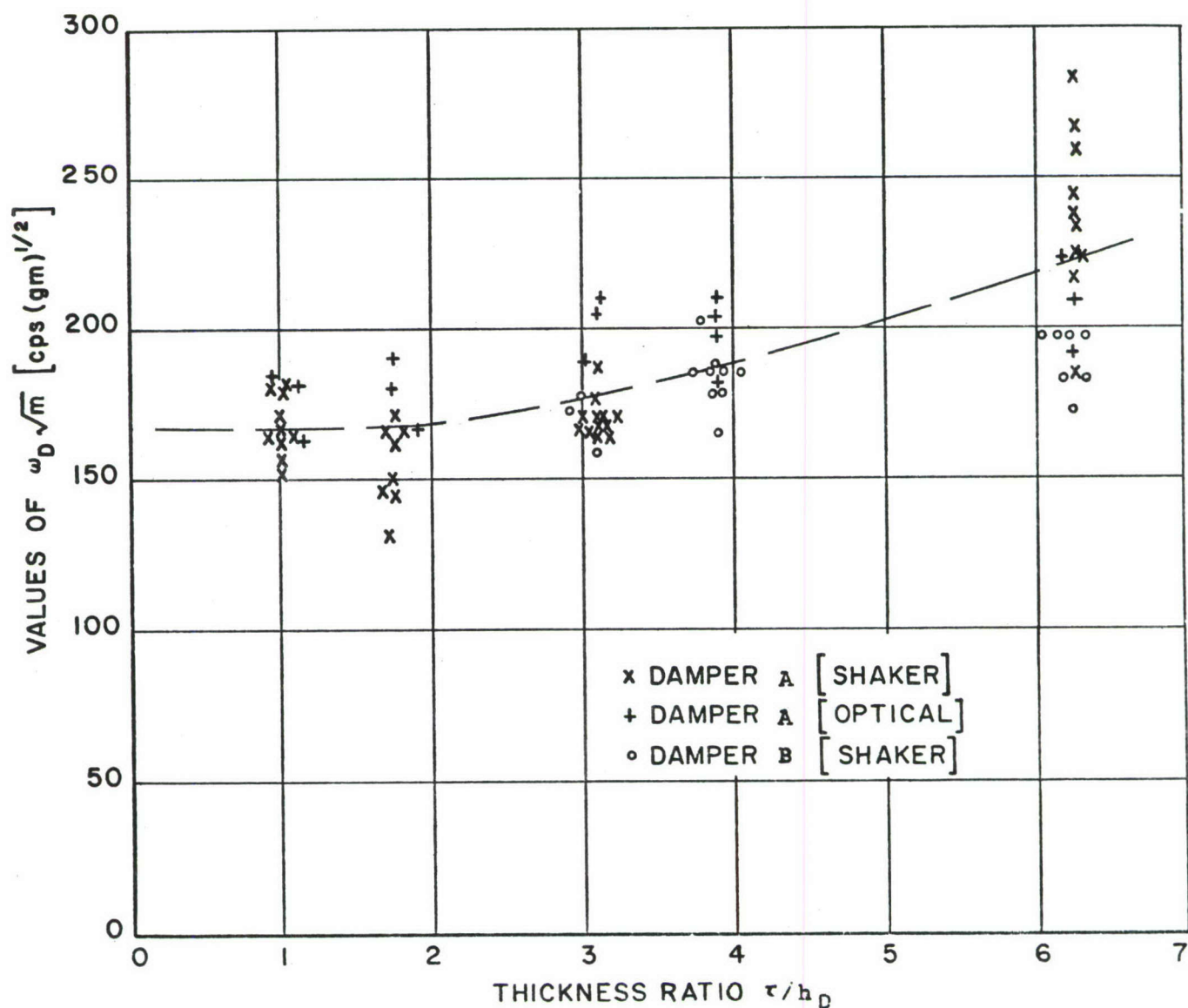


Fig 16. Graph of $\omega_D m^{1/2}$ Against τ/h_D for Tuned Dampers Used on Cantilever Beams (Both Distributed and Singly at Free End)

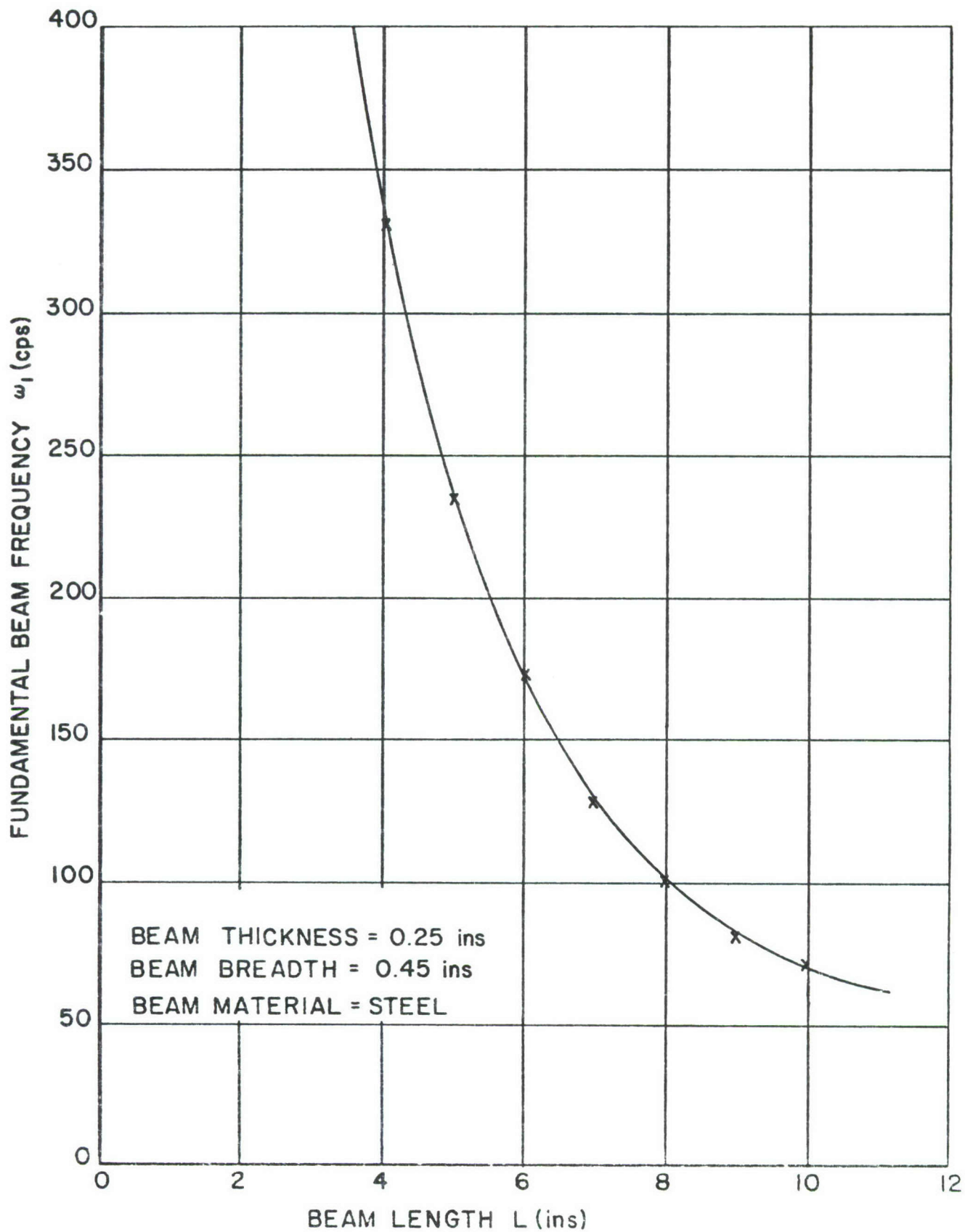


Figure 17. Graph of Fundamental Frequency ω_1 of Cantilever Beam
Against Length L

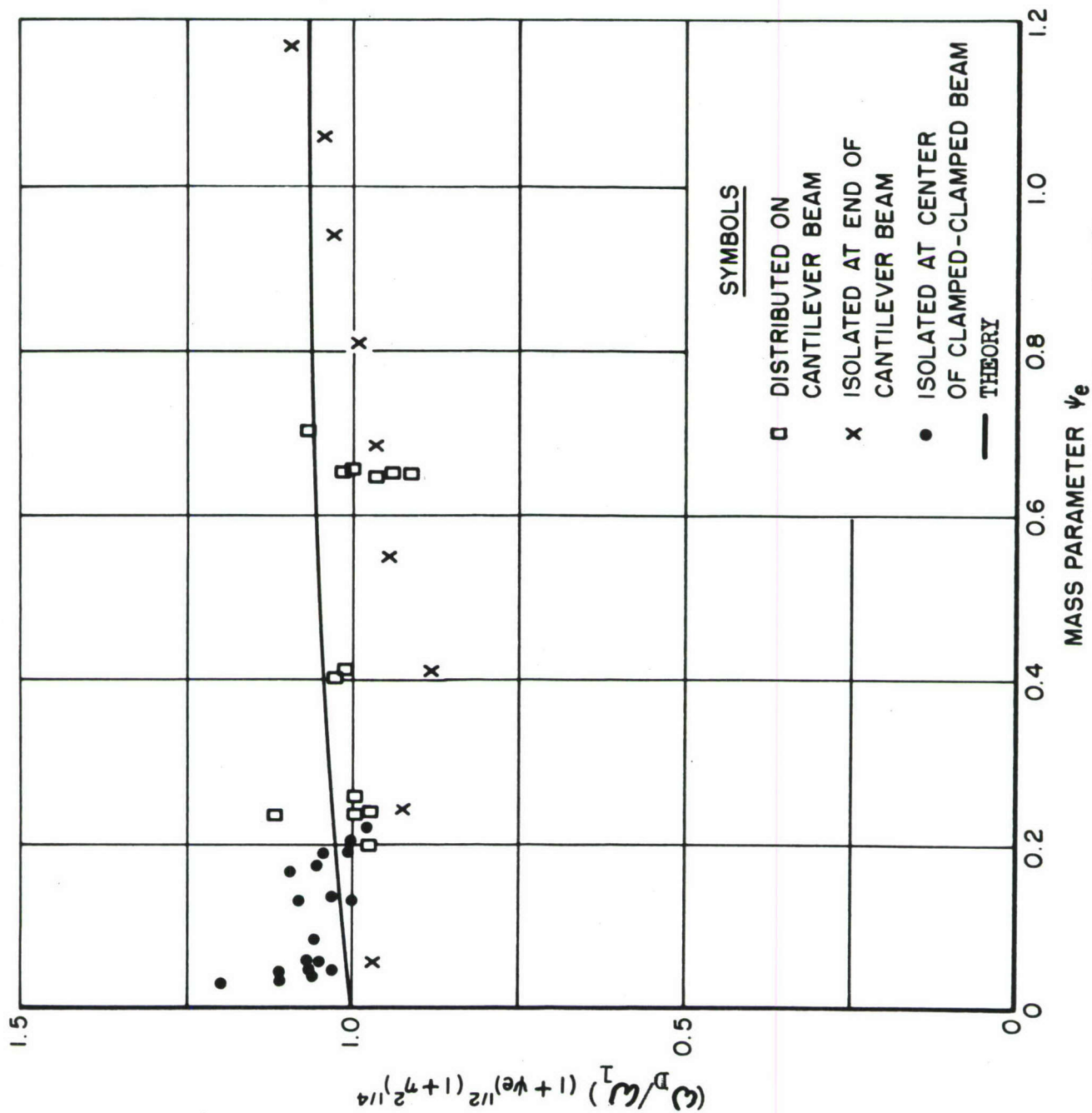


Figure 18. Experimental values of $(\omega_D/\omega_1)(1+\psi_e)^{1/2}(1+\eta^2)^{1/4}$ Plotted Against ψ_e

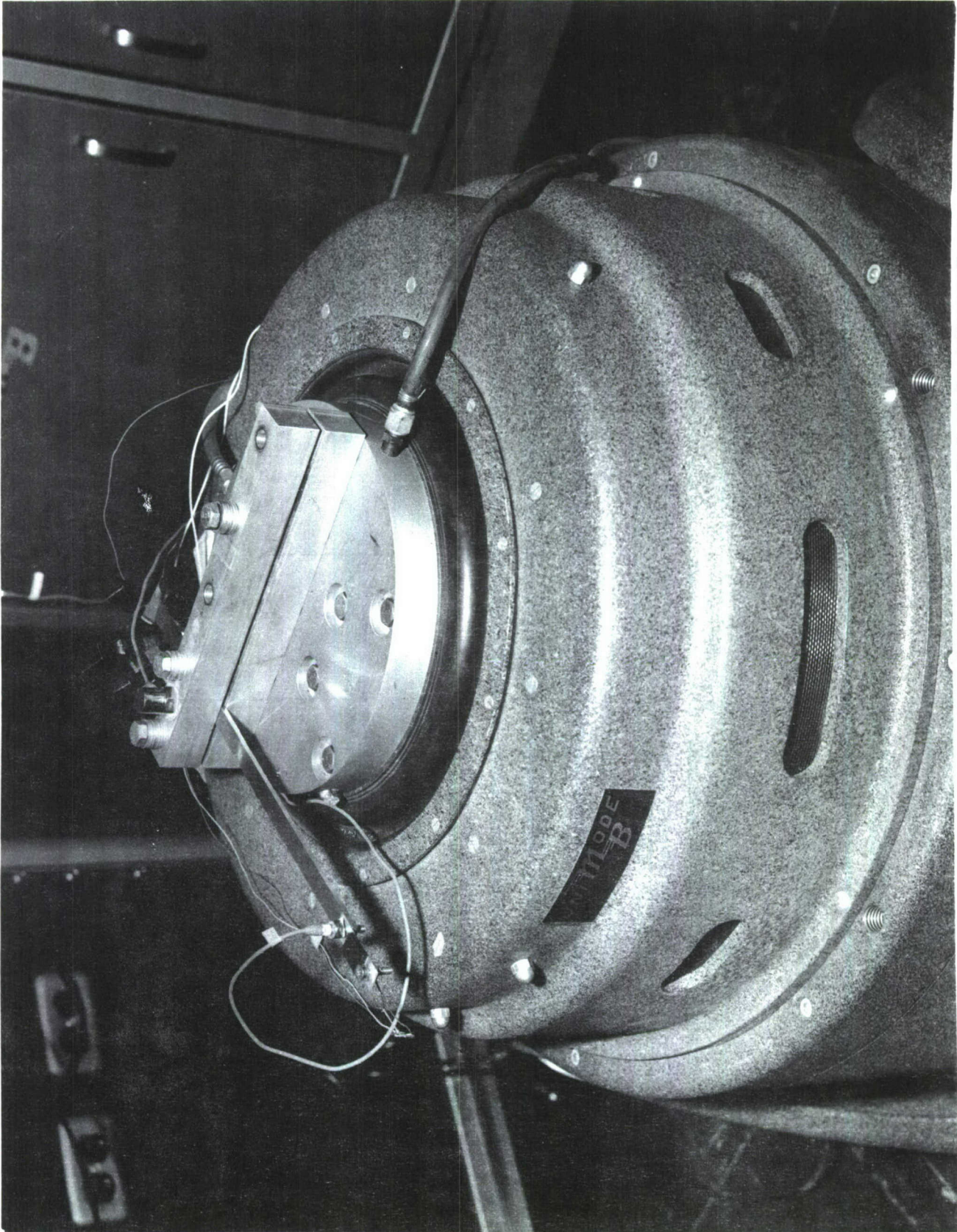


Figure 19. Photograph of Tuned Damper at Free End of Cantilever Beam

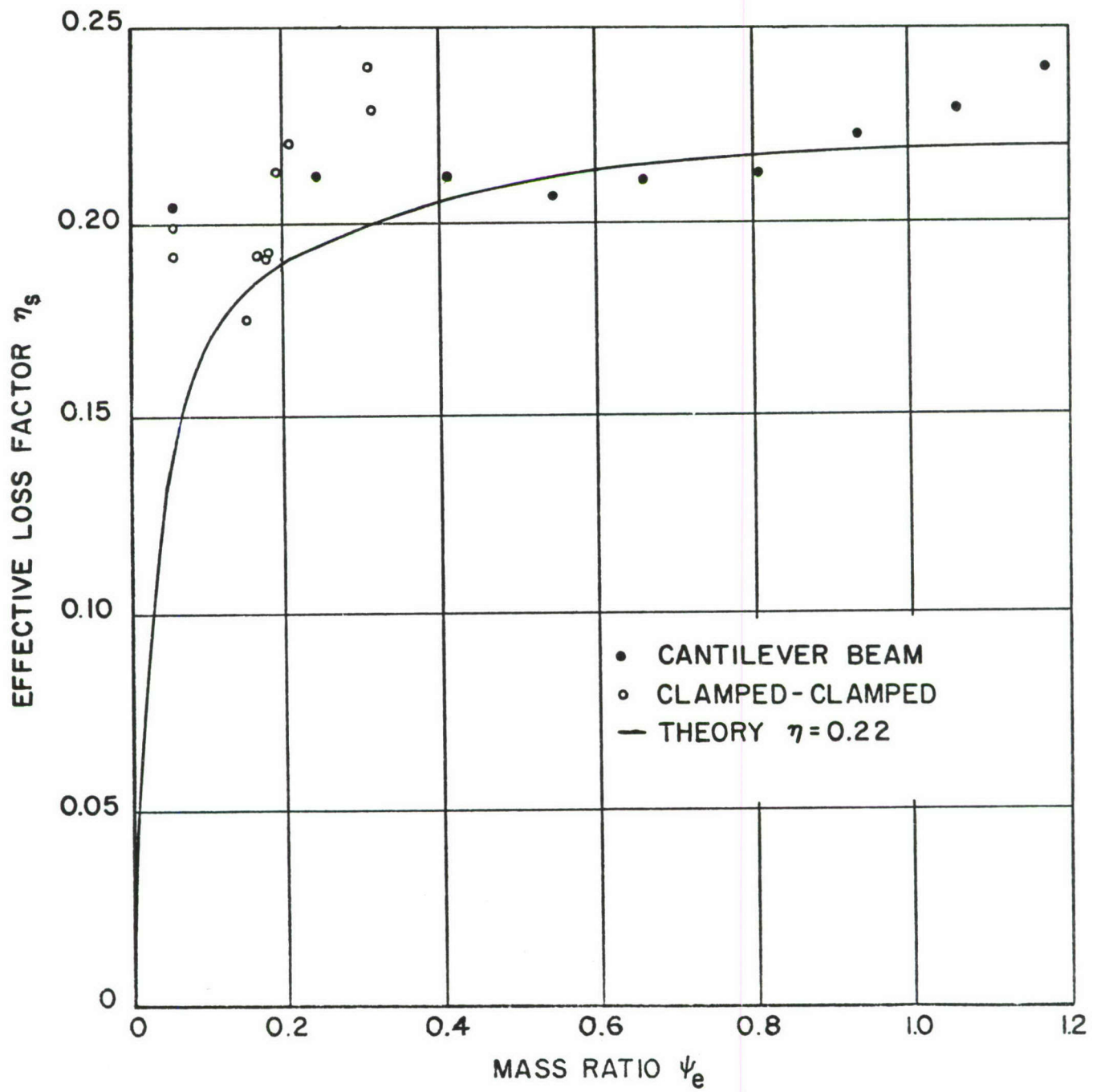


Figure 20. Experimental Values of η_s Plotted Against ψ_e

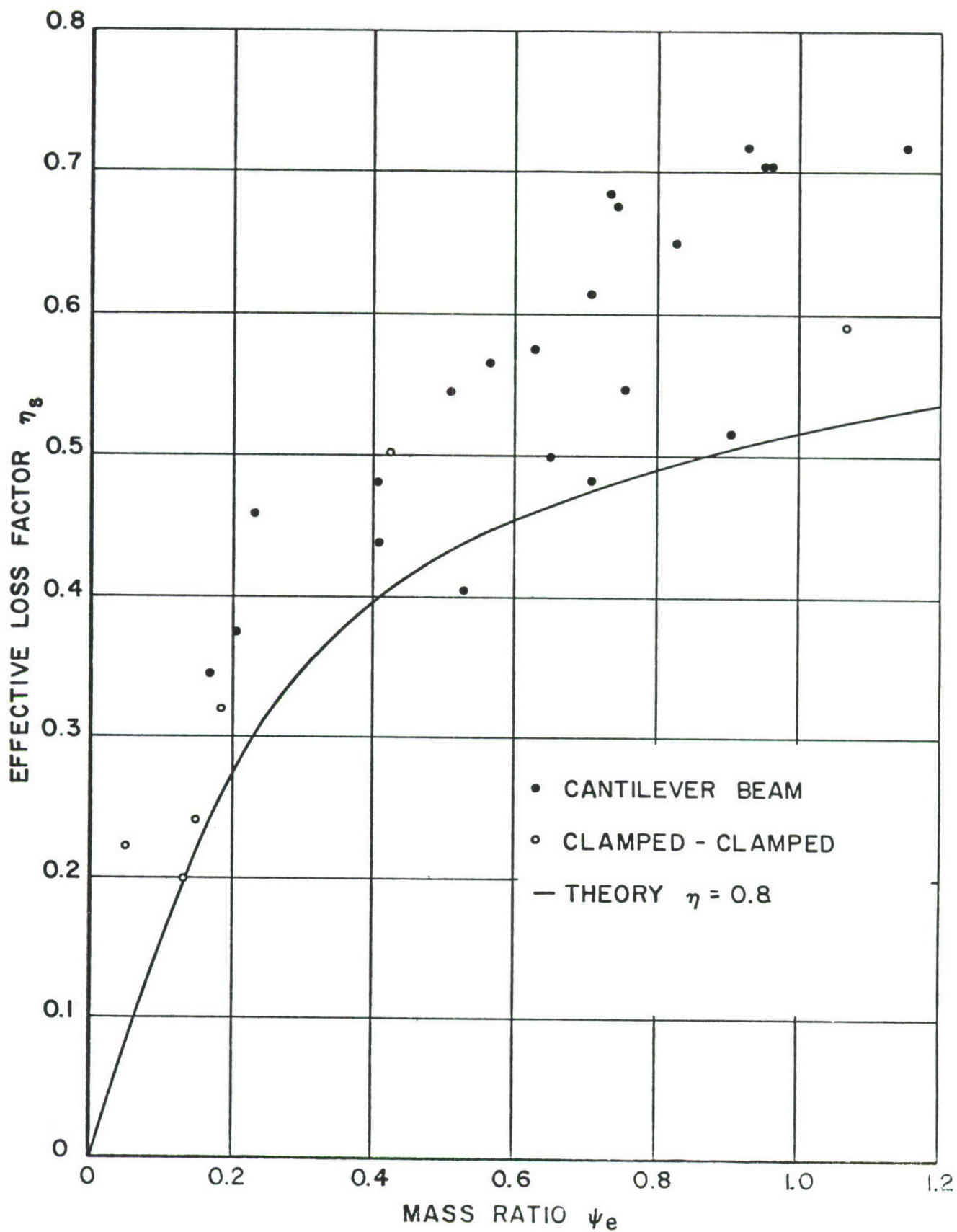


Figure 21. Experimental Values of η_s Plotted Against ψ_e

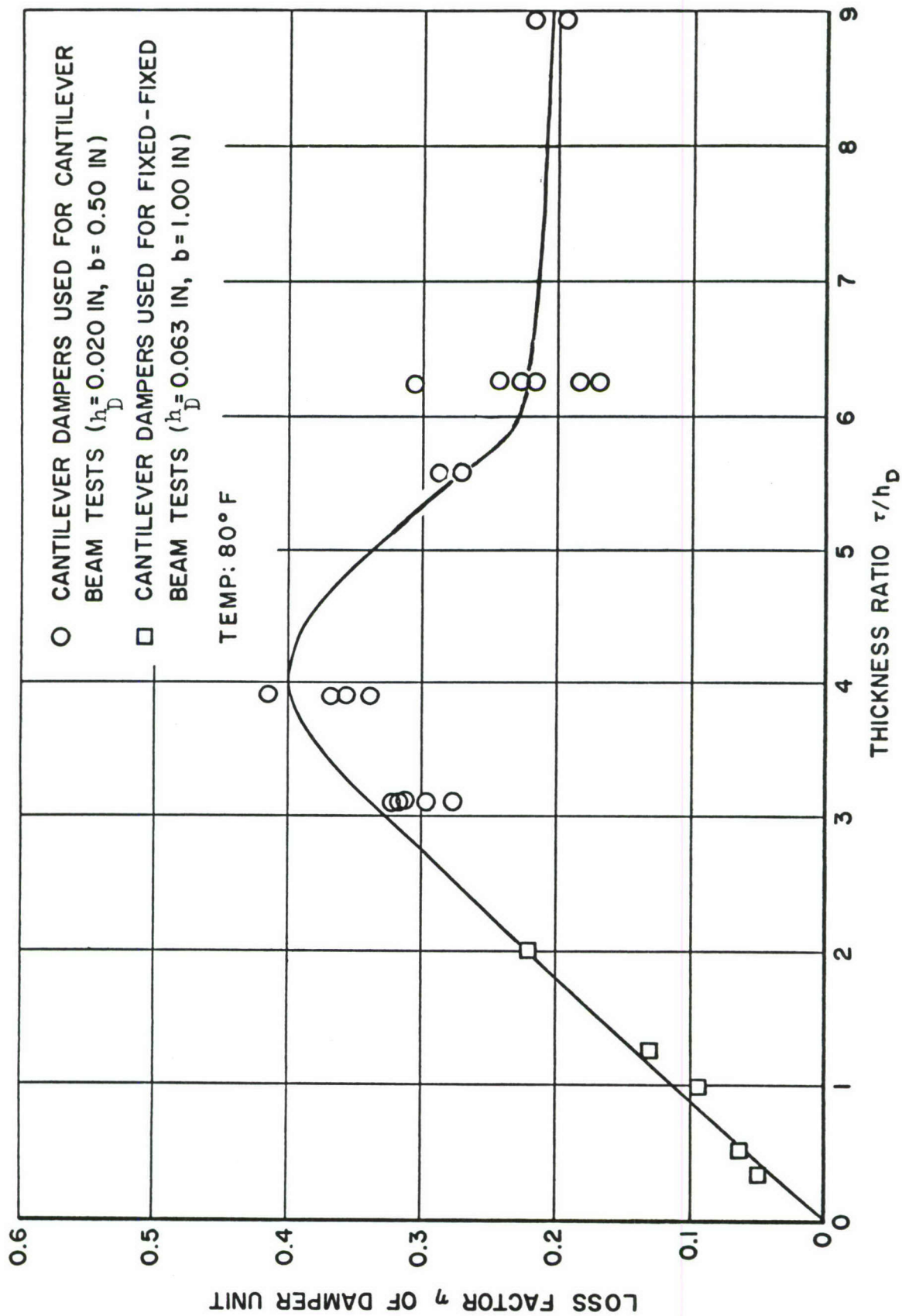


Figure 22. Graph of cantilever damper loss factor η against ratio of viscoelastic layer thickness to thickness of metal cantilever (80°F.)

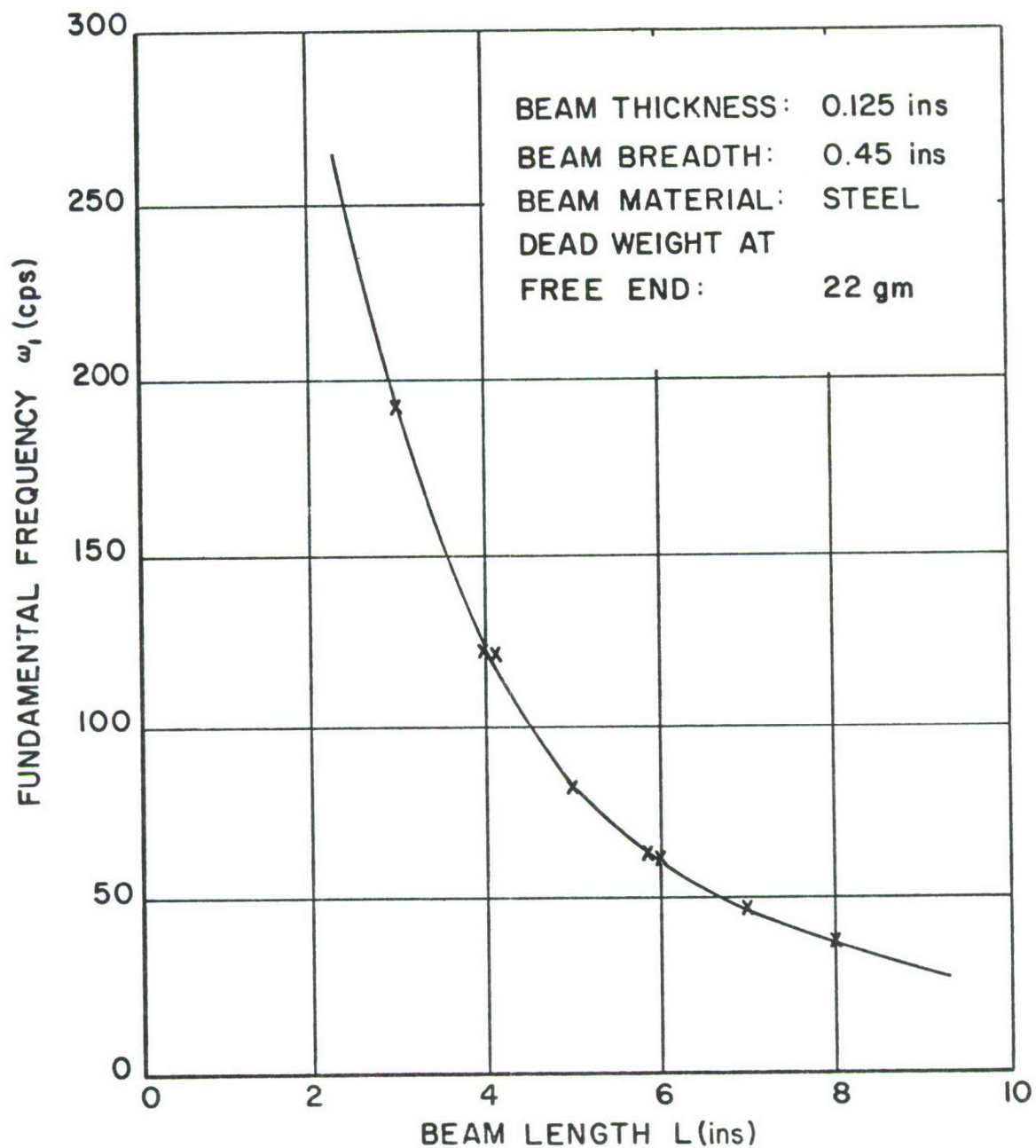


Figure 23. Graph of Fundamental Frequency ω_1 of Cantilever Beam
Against Length L (Dead Weight of 22 gm at Free End)

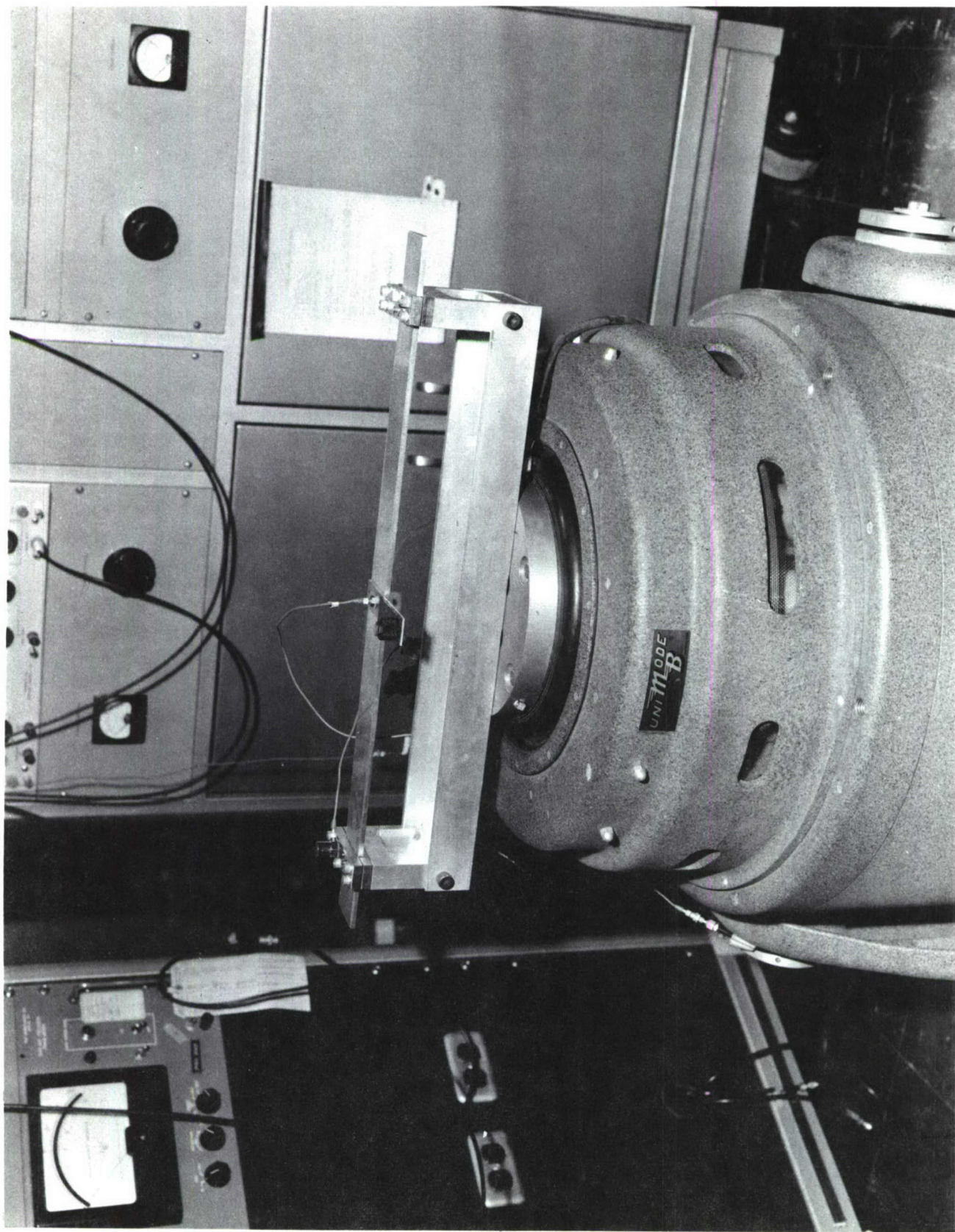


Figure 24. Photograph of Cantilever Damper on Clamped - Clamped Beam

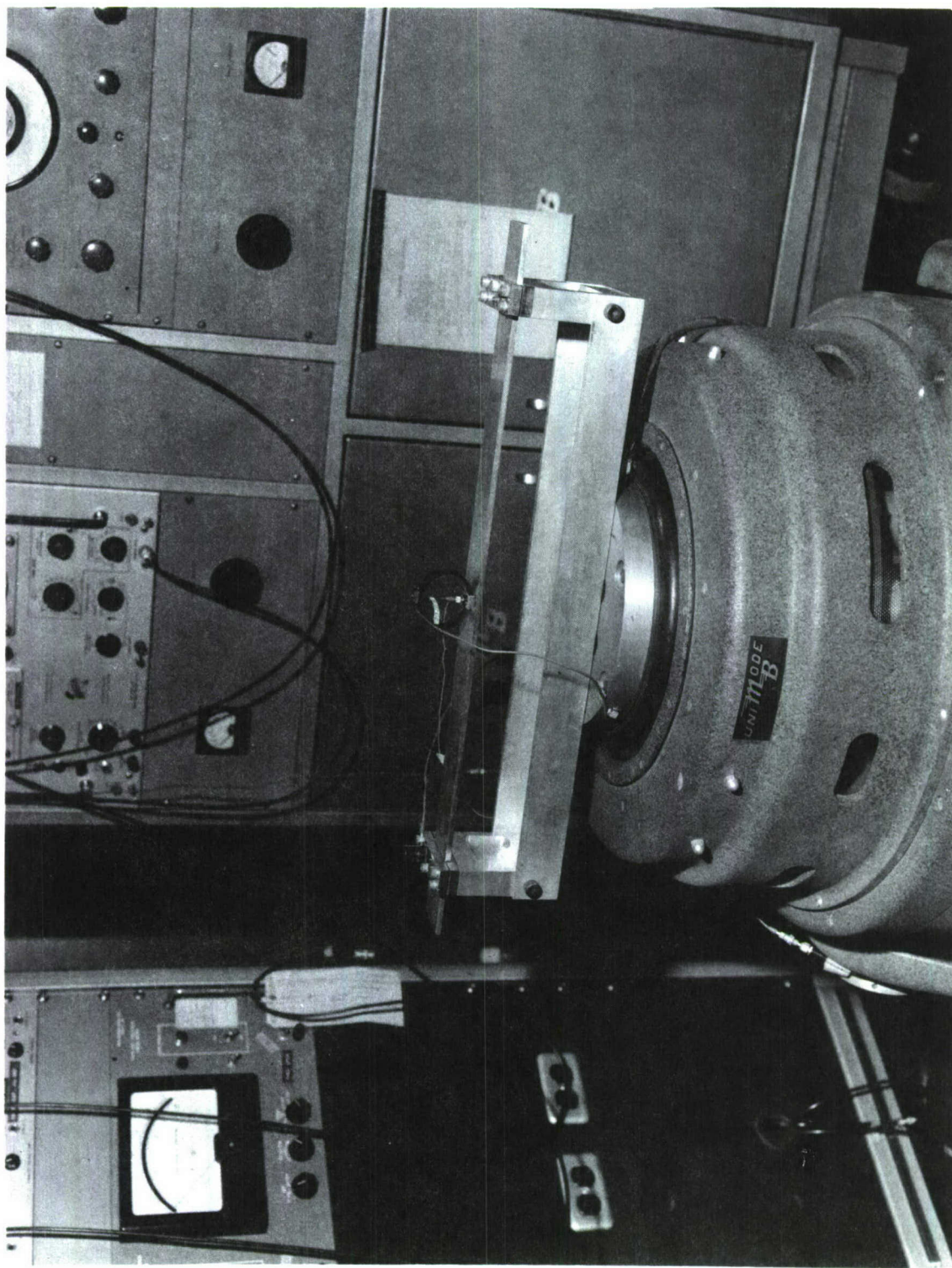


Figure 25. Photograph of Ring Damper on Clamped-Clamped Beam

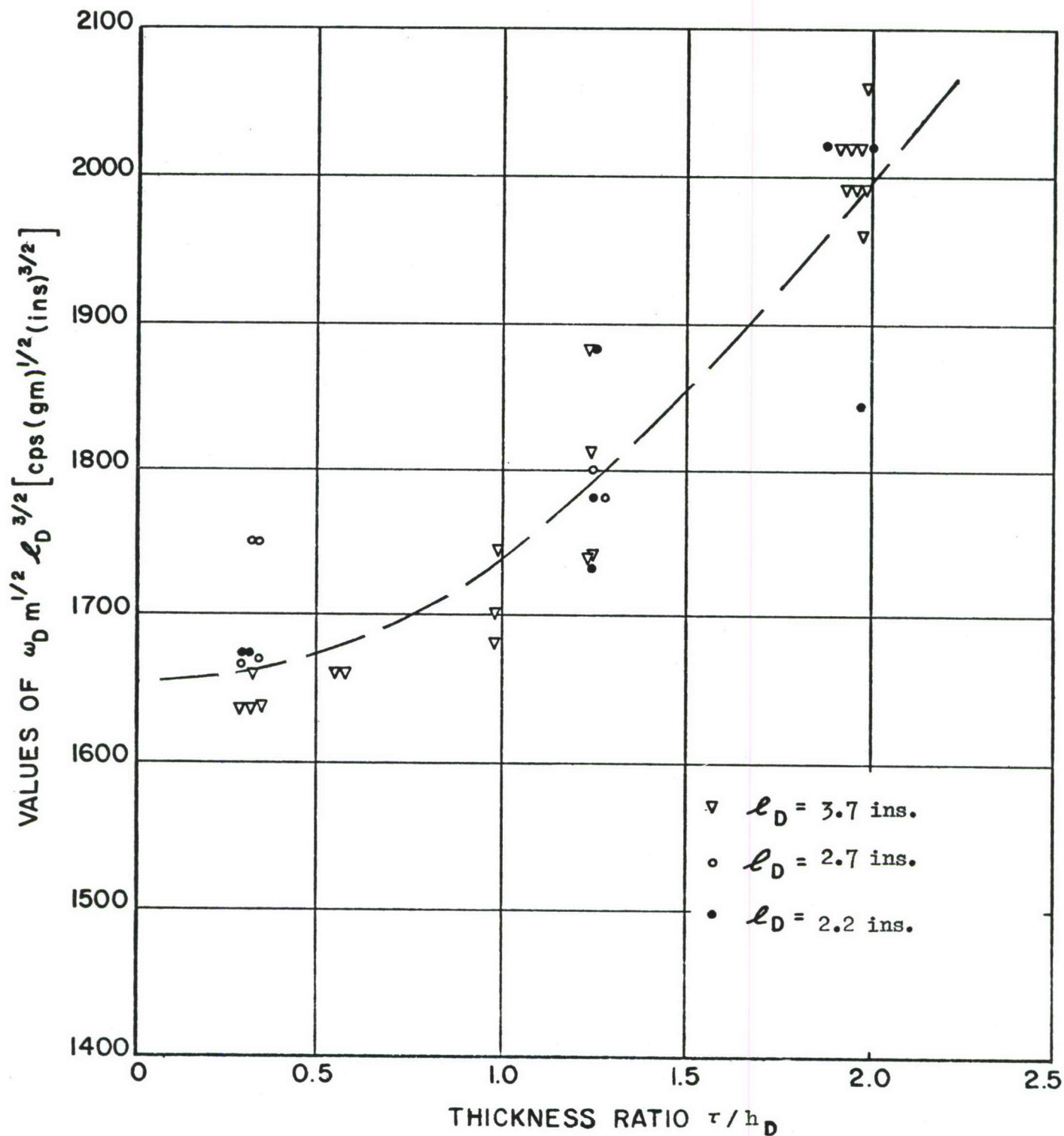


Figure 26. Graph of $\omega_D m^{1/2} l_D^{3/2}$ Against τ/h_D for Tuned Dampers Used On Clamped-Clamped Beam

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4. DESCRIPTIVE NOTES (Type of report and inclusive dates) Summary Report - February 1965 to November 1966		
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13. ABSTRACT <p>An approximate analysis of the response of the fundamental mode of any simple single span beam with tuned viscoelastic dampers attached at discrete locations to a harmonic loading with arbitrary spatial distribution is derived. It is shown that, to a good degree of approximation, a single expression can be made to represent the response in the fundamental mode of a beam with any boundary conditions, provided that certain effective mass and stiffness parameters are defined for the beam-damper configuration. Comparisons are made with experiments and with an exact theory, subject to the limitations of the Euler-Bernoulli beam equation, of the response and damping of a cantilever beam having an isolated harmonically varying load at the free end and a clamped-clamped beam, with a tuned damper at the center, under shaker excitation. Good agreement between the exact and approximate theories and the experiments is demonstrated. Conclusions are drawn concerning the equivalent damping introduced into the simple structure by the tuned dampers and the damper natural frequency needed for optimal damping.</p> <p>This abstract is subject to special export controls.</p>		

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